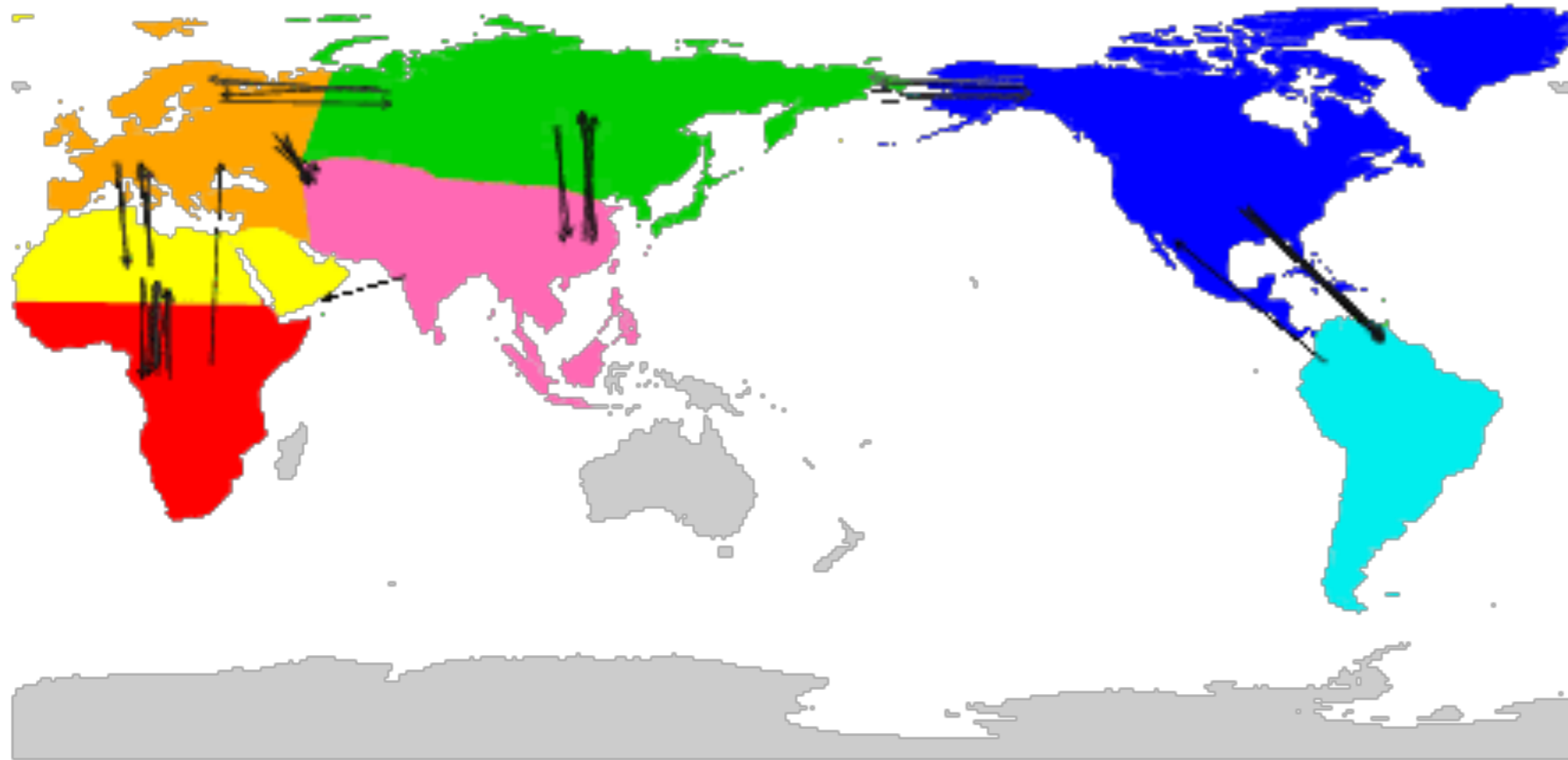


Phylogenetic biogeography of fossil Canidae: combining models of imperfect detection, databased fossils as taphonomic controls, and Biogeographical Stochastic Mapping



Nicholas J. Matzke
DECRA Fellow

Division of Ecology, Evolution, and Genetics
Research School of Biology
The Australian National University

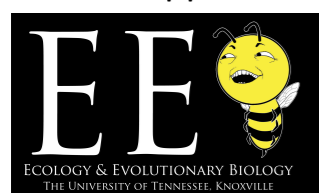
phylo.wikidot.com/nicholas-j-matzke

23 June, 2017, 4 p.m.

Session: Biogeography / phylogeography
Evolution 2017, Room A106

possible
histories
(DEC+j+w
model)

Thanks for support from:



Collaborator: Laura K. Säilä
Dept. of Geosciences and Geography
University of Helsinki, Finland



Acknowledgements

Questions/comments/
collaborations at:
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Katie Massana

Michael Landis

Ph.D. committee

John Huelsenbeck

Tony Barnosky

David Jablonski

Roger Byrne

PhyloWiki
Assisting research and education in phylogenetics and evolution

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What is a Wiki Site?
How to edit pages?
How to join this site?
Site members
Recent changes
List all pages
Page Tags
Site Manager

Page tags
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free icons
Add a new page
new page
edit this panel

BioGeoBEARS

Fold

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- Update May 2014: Stochastic mapping
- Update March 2014: Simulation results
- Update to simulation results: Animations
- Update February 2014: Manual construction of BioGeoBEARS legends
- Updates February 2014: Fixing states at multiple nodes.
- Updates January 2014: Fixes for optimx2012/optimx2013/optim issues; bug fix in uppass calculations
- Update December 2, 2013: Citations for BioGeoBEARS and associated models
 - Citation of R packages
- Update November 12, 2013: BioGeoBEARS in one slide
- Update November 8, 2013: Using time-stratification, dispersal multiplier matrices, distance matrices, areas allowed matrices, and area-of-areas matrices
- Update October 1, 2013: Listserv is up!
- Update September 10, 2013: Interested in listserv?
- Update August 24, 2013: issue with "optimx" dependency (resolved Jan. 2014)
- Introduction
- Installation
- Updates
- SCRIPT: EXAMPLE THAT SHOULD RUN OUT OF THE BOX: Hawaiian Psychotria (revised and improved, 2014-03-21)
- SCRIPT: BioGeoBEARS legends

See also: Publications using BioGeoBEARS.

TRY BioGeoBEARS AT:

<http://phylo.wikidot.com/biogeobears>

Funding: NIMBioS

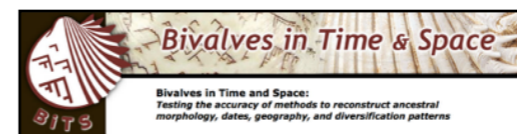
NSF "Bivalves in Time and Space"

UC Berkeley Wang Fellowship

UC Berkeley Tien Fellowship

Google Summer of Code

NIMBioS, ARC DECRA



Family Canidae

~35 living species

~120+ fossil species

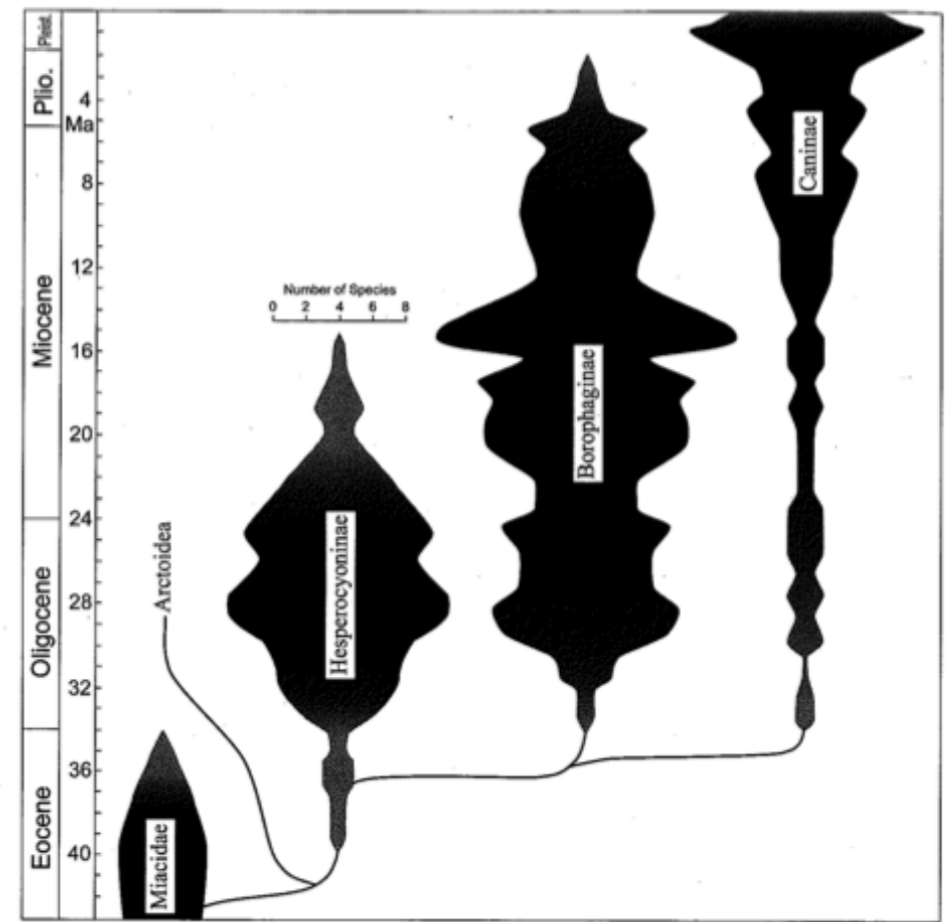
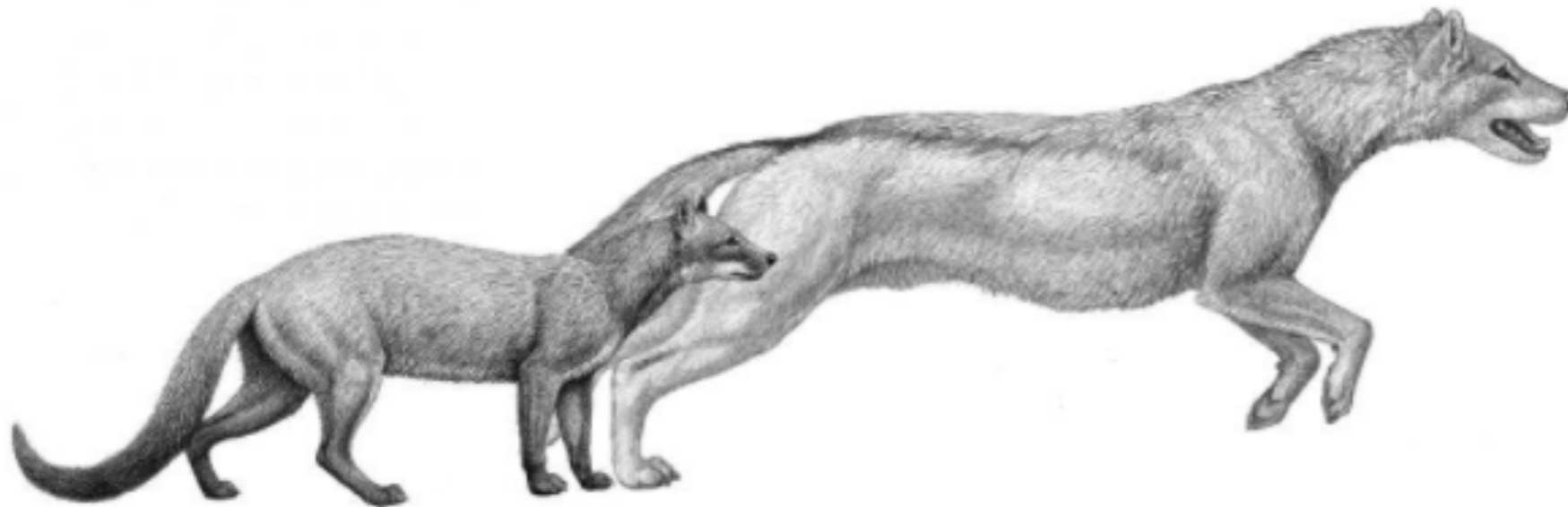


FIGURE 6.6

Diversity of species through time in the three subfamilies of Canidae

(Wang & Tedford 2008)



Hesperocyon and Sunkahetanka, two species of early dog. They were quite small and more closely resembled a mongoose.

(Illustration by Mauricio Anton)

Source: <http://www.forbes.com/sites/shaenamontanari/2015/08/18/old-dogs-learned-new-tricks-for-hunting-as-climate-changed/>

Fossil Canidae

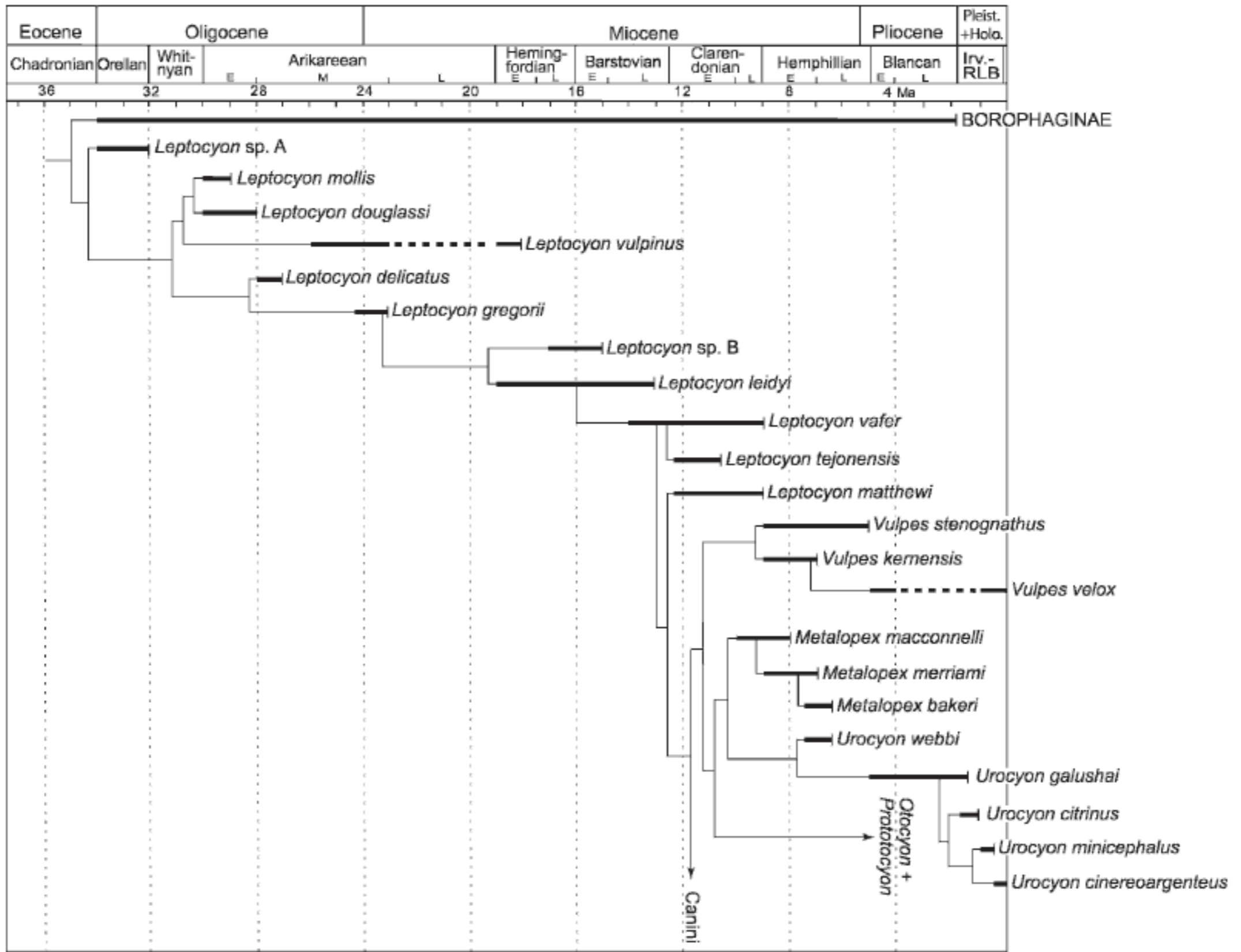
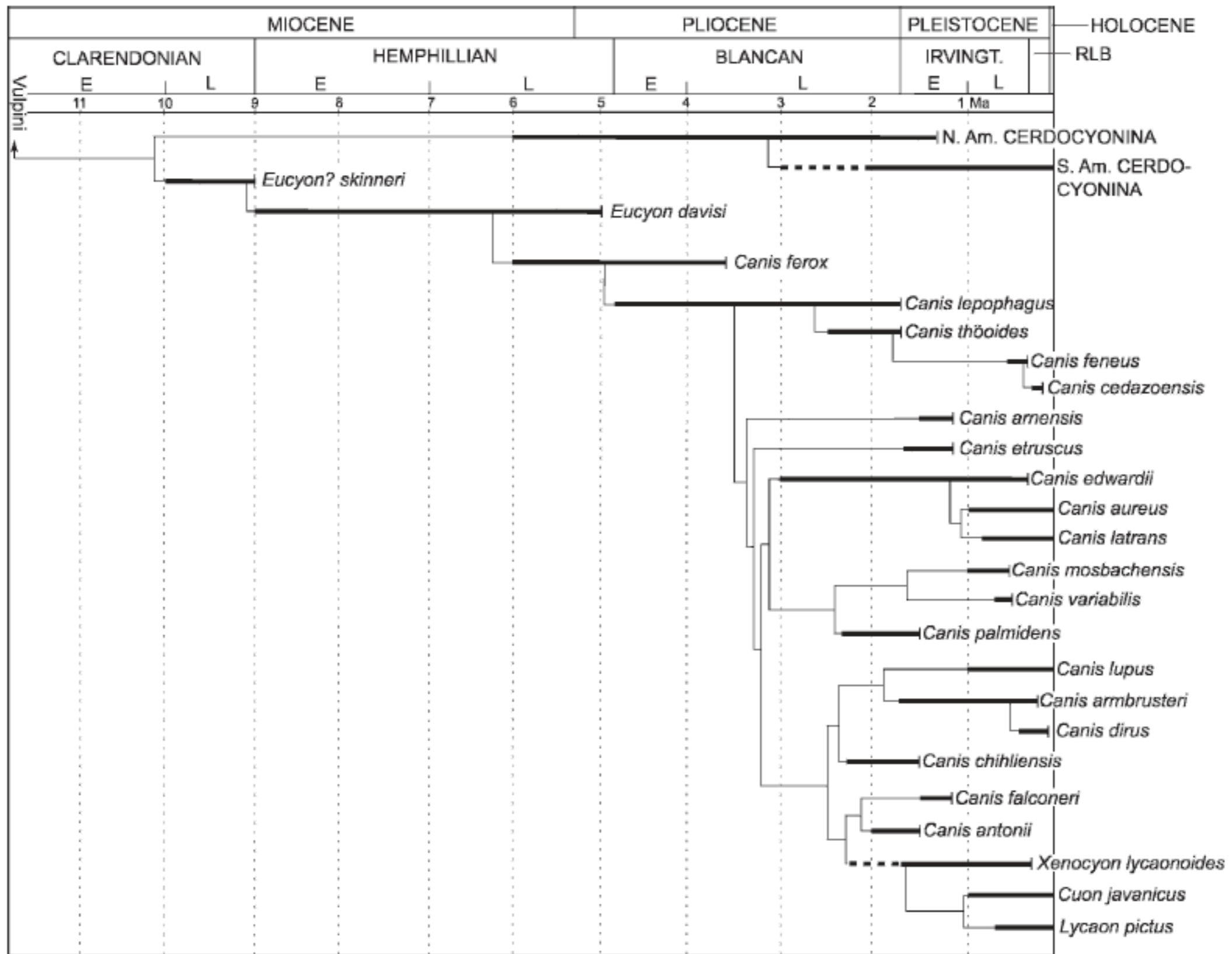


Fig. 66. Stratigraphic ranges and postulated phyletic relationships for taxa of the subfamily Caninae discussed in this report. The relationship is largely based on the preferred cladogram in figure 65, modified by speculations about cladogenetic or anagenetic events. These speculations are based on morphologic features too subtle to be coded in the data matrix but which, nonetheless, offer clues about possible relationships.

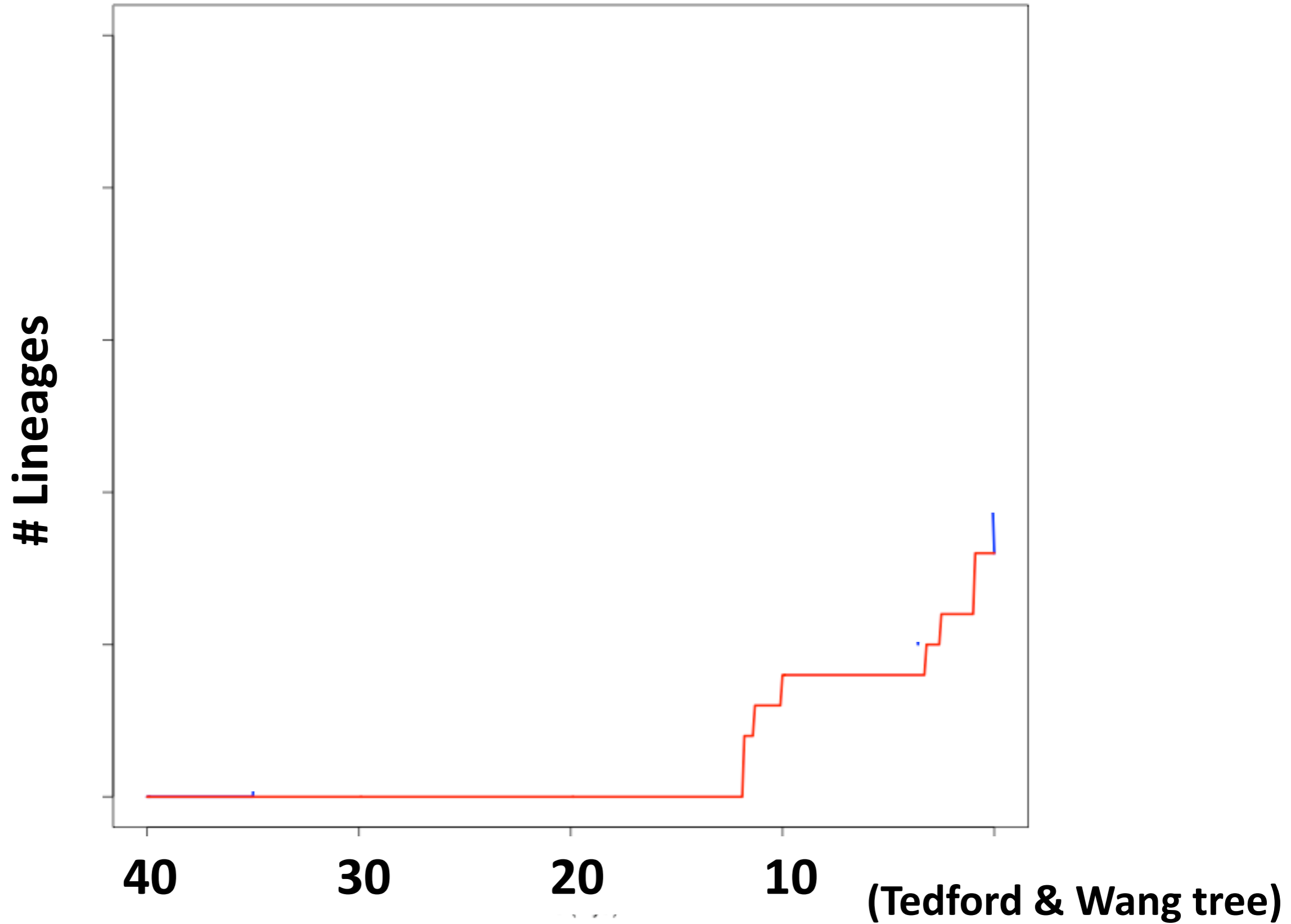
Fossil Canidae

Fig. 66. *Continued.* Taxa that are too poorly known to be included in the phylogenetic analysis are also plotted on this chart. Chronologic framework is based on Tedford et al. (2004). Geographic range changes of Cerdocyonina are shown that schematically reflect dispersal to South America and extinction in North America.



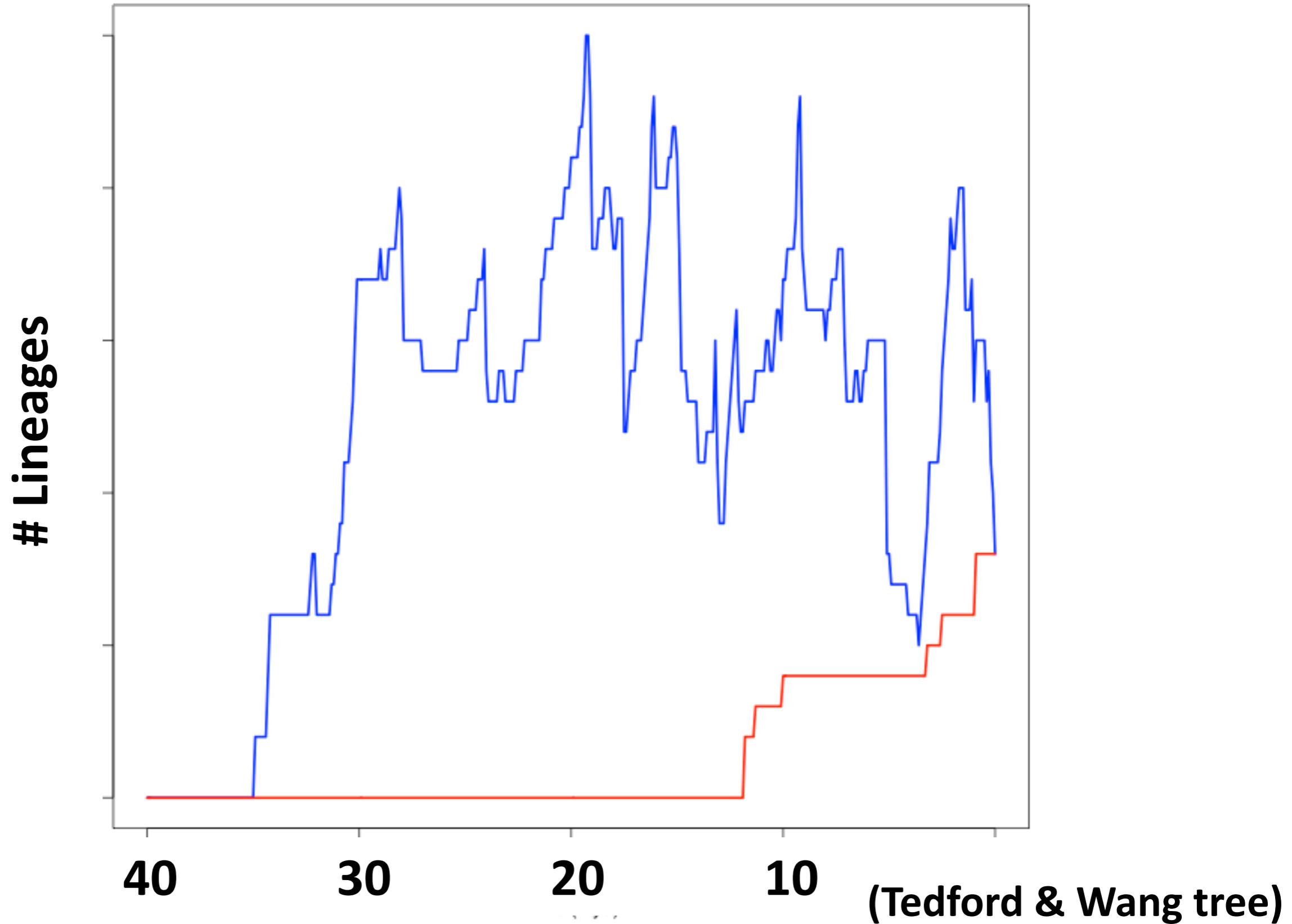
Family Canidae

(minimum) Lineages-through-time plot



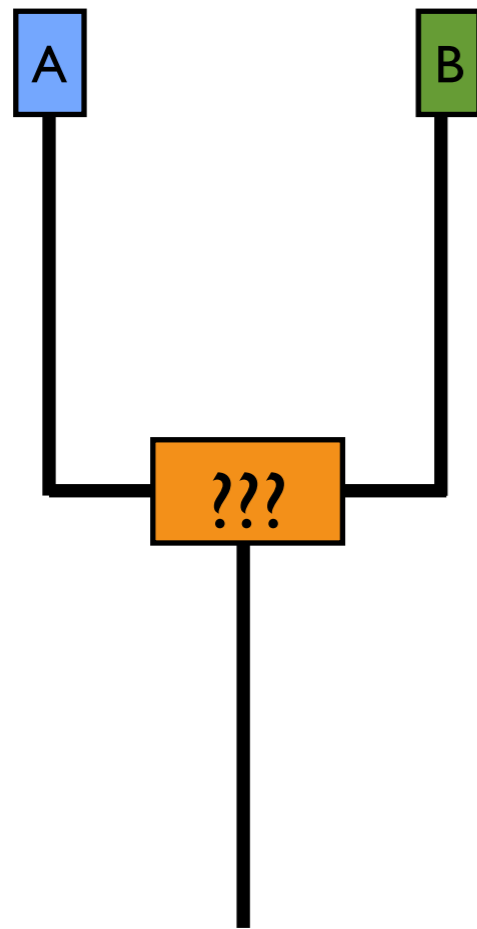
Family Canidae

Lineages-through-time plot with fossils



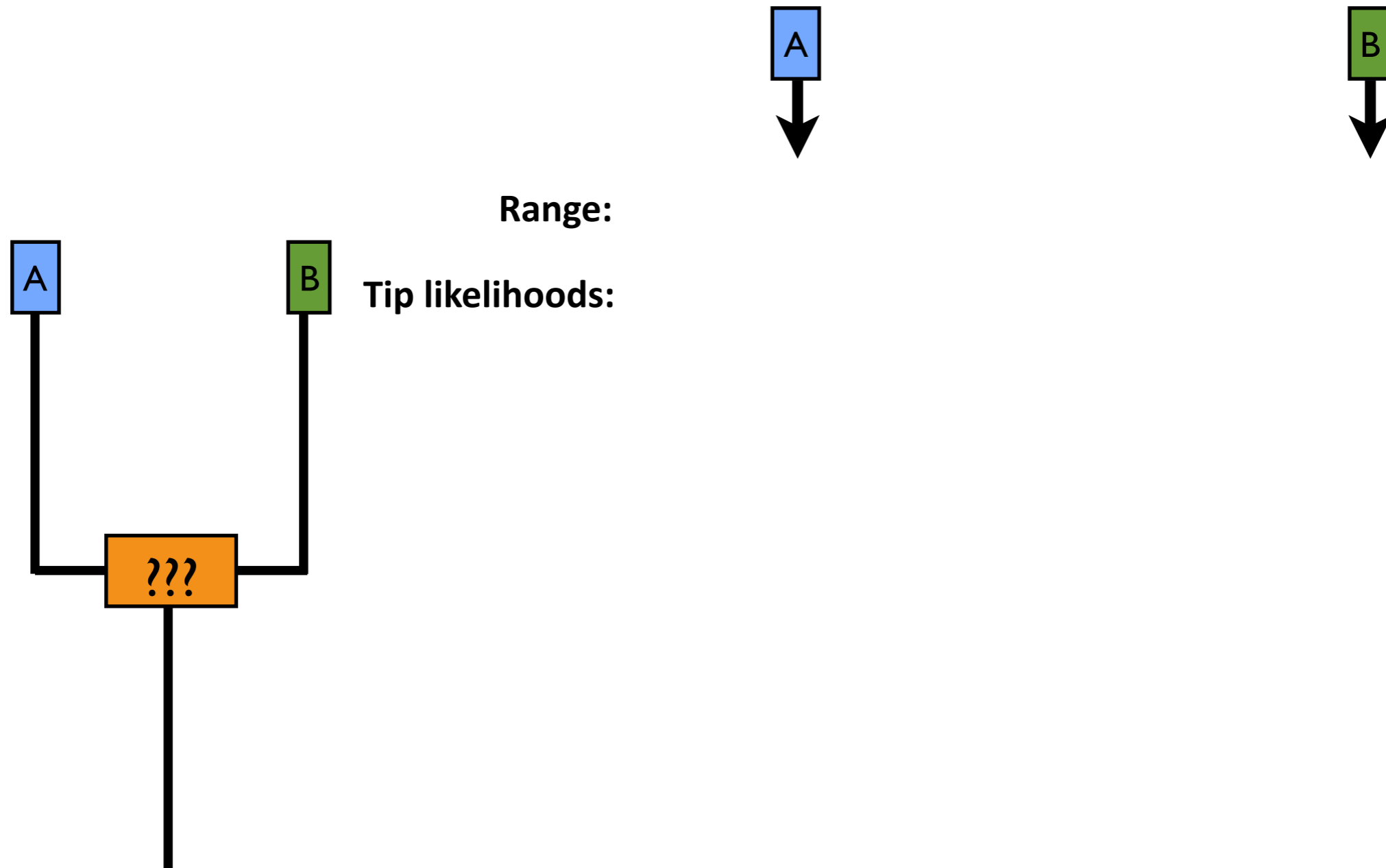
Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Strategy #1. Constraints on node ranges



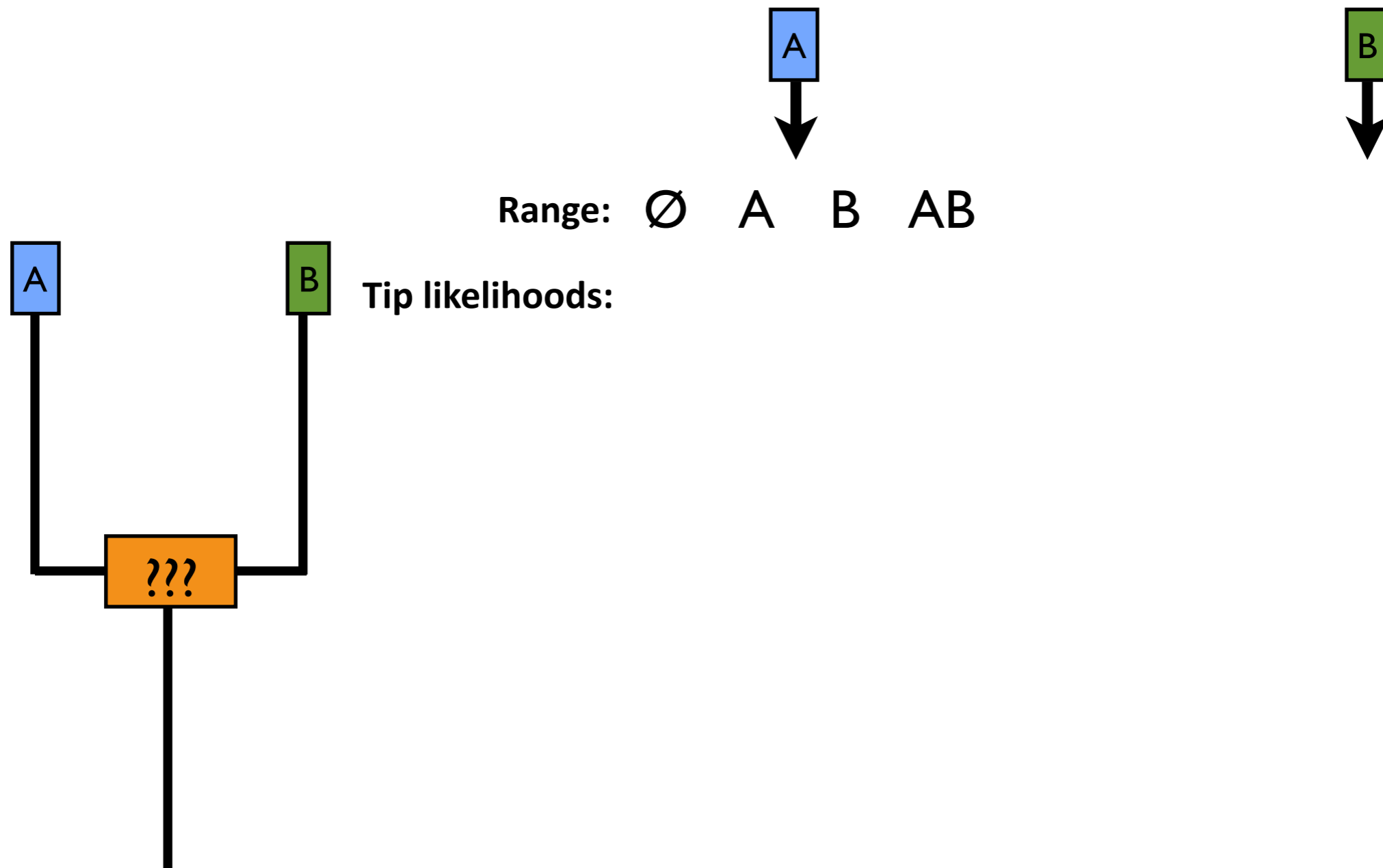
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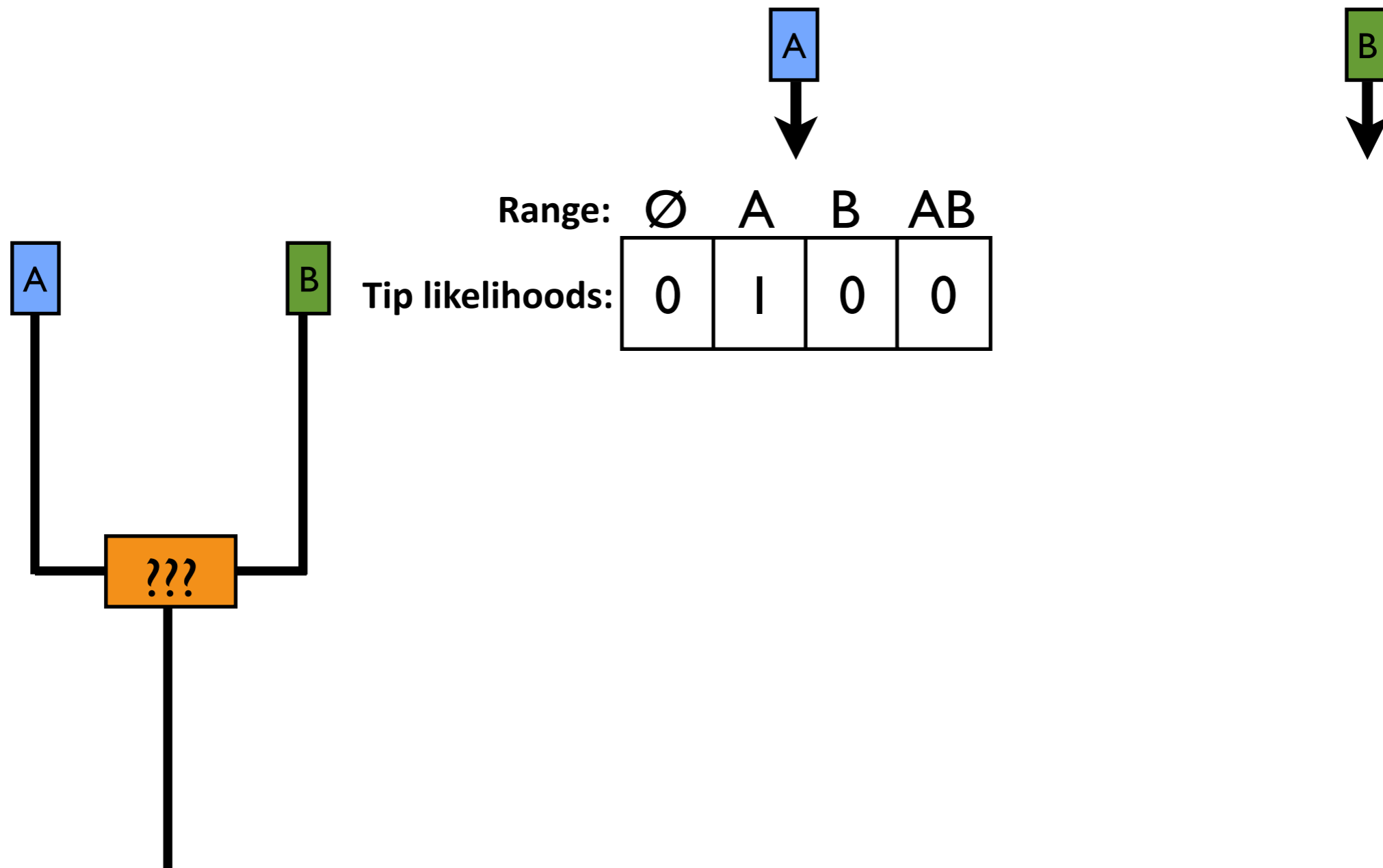
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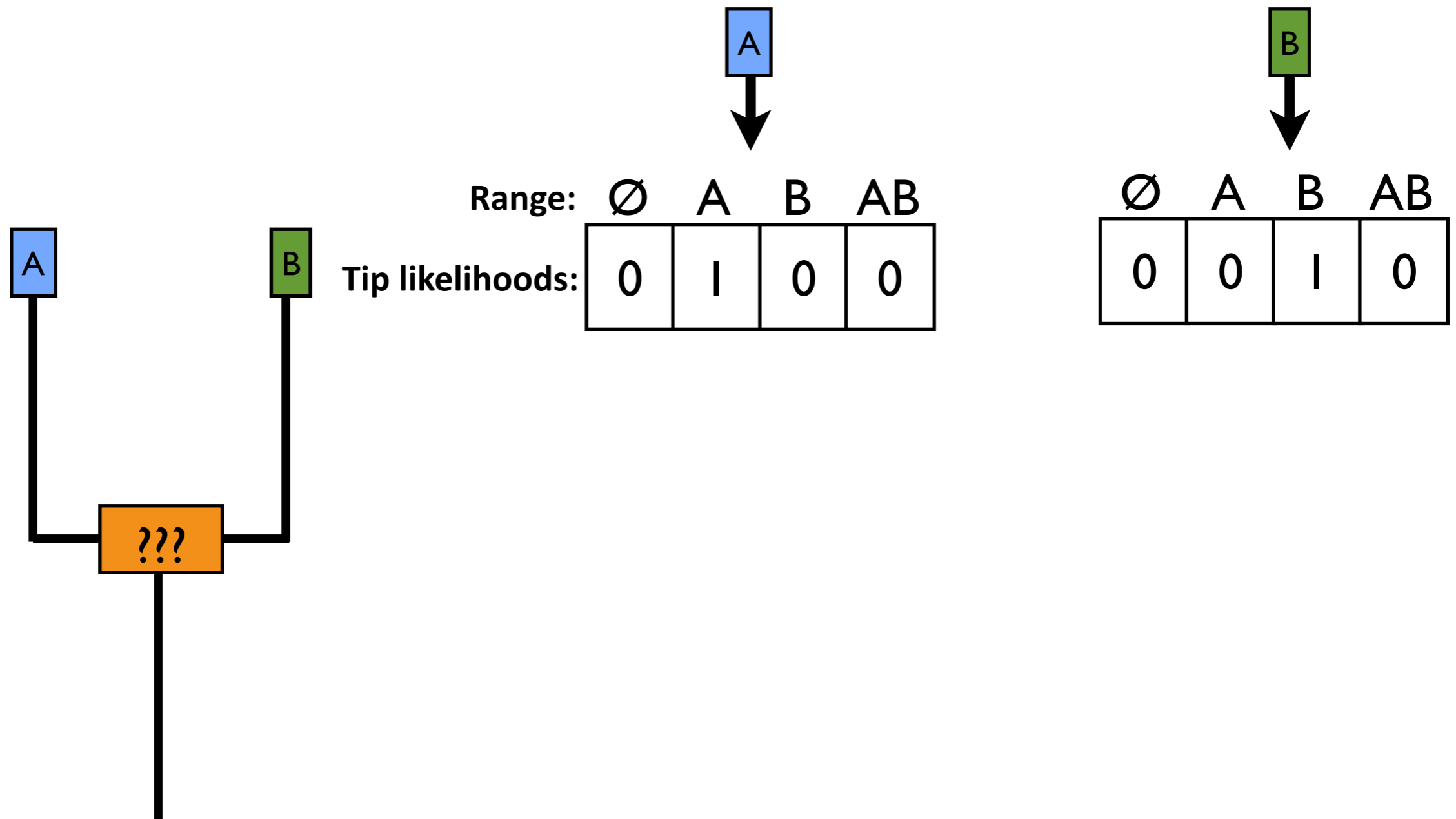
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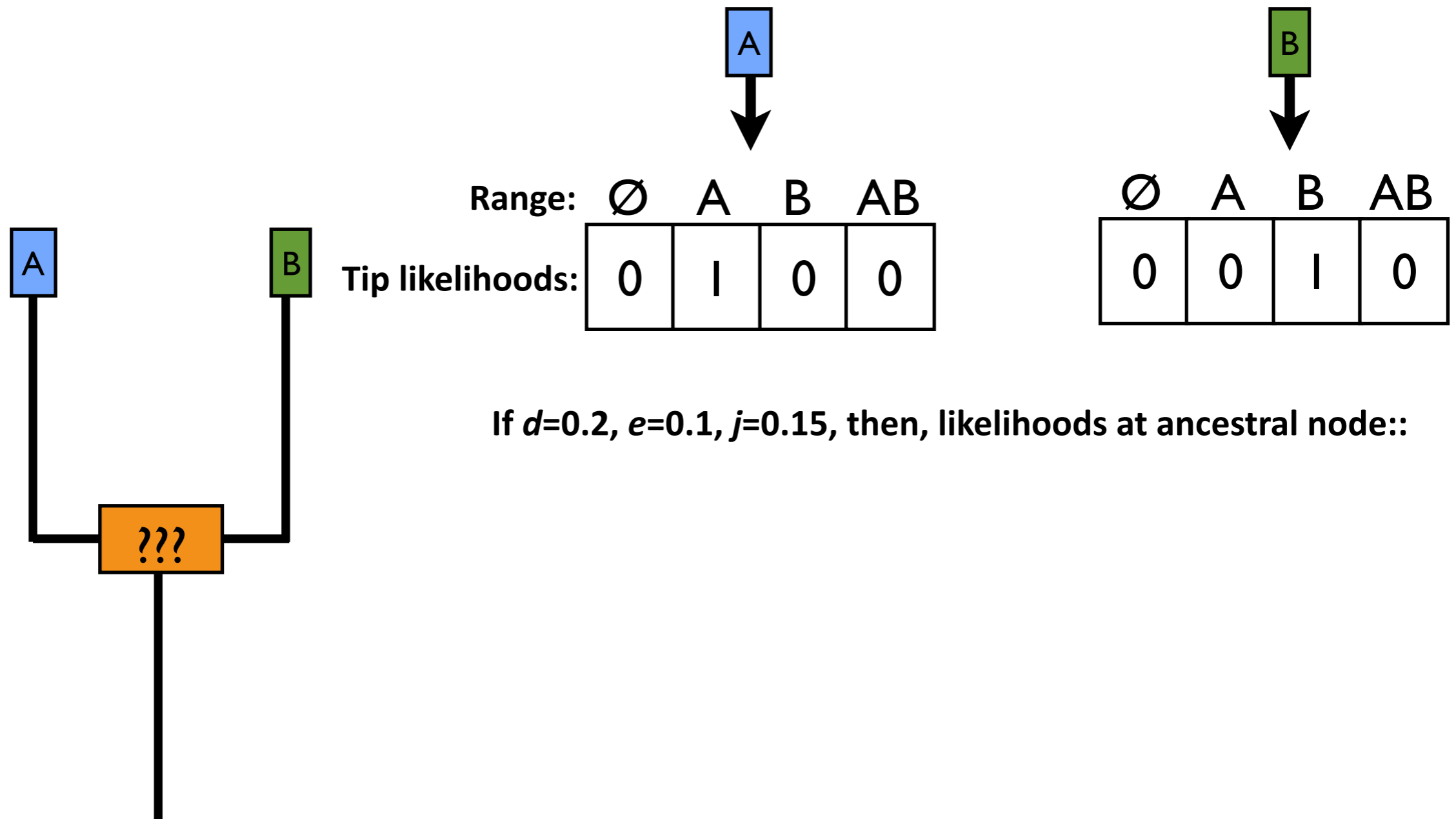
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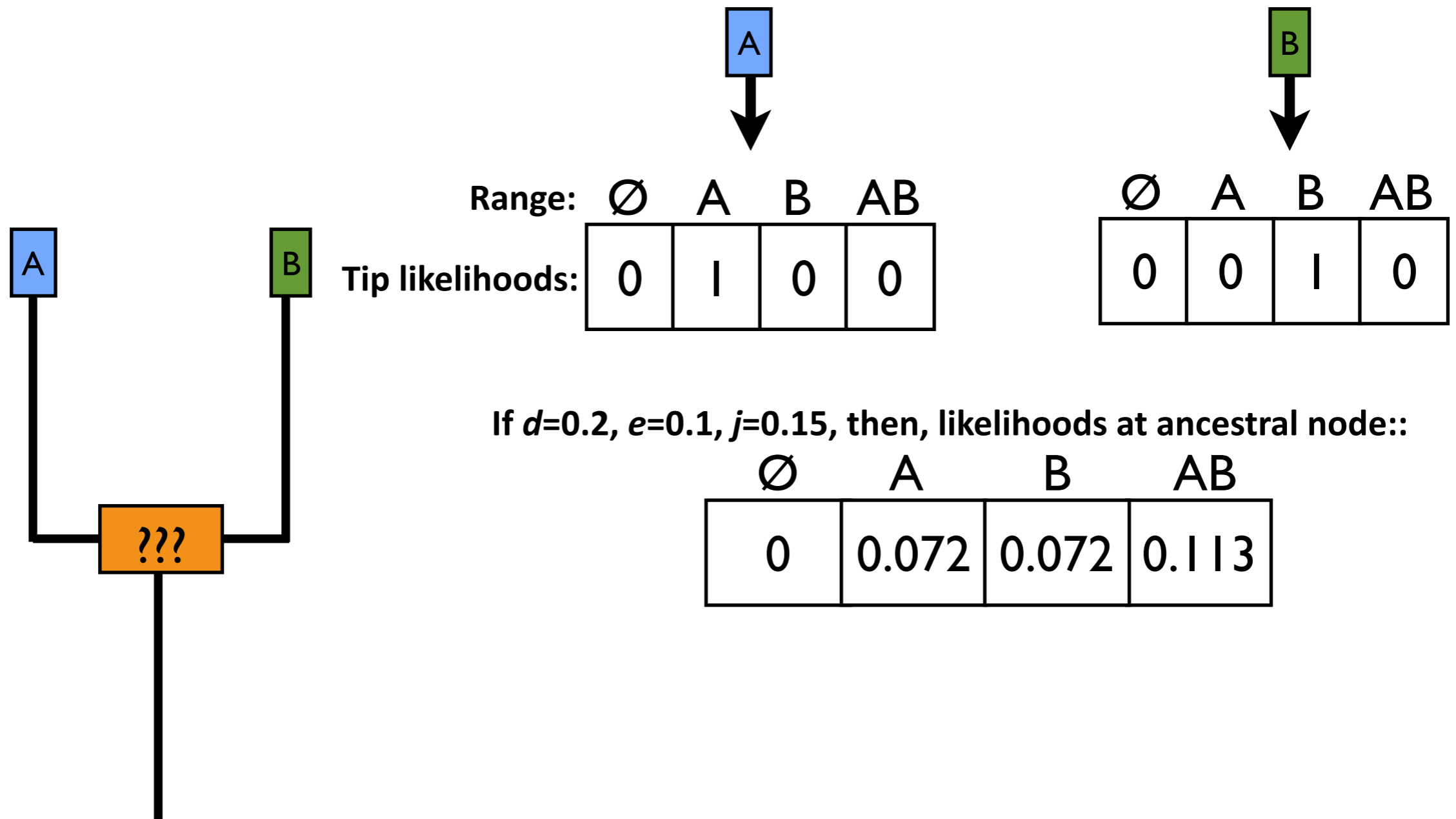
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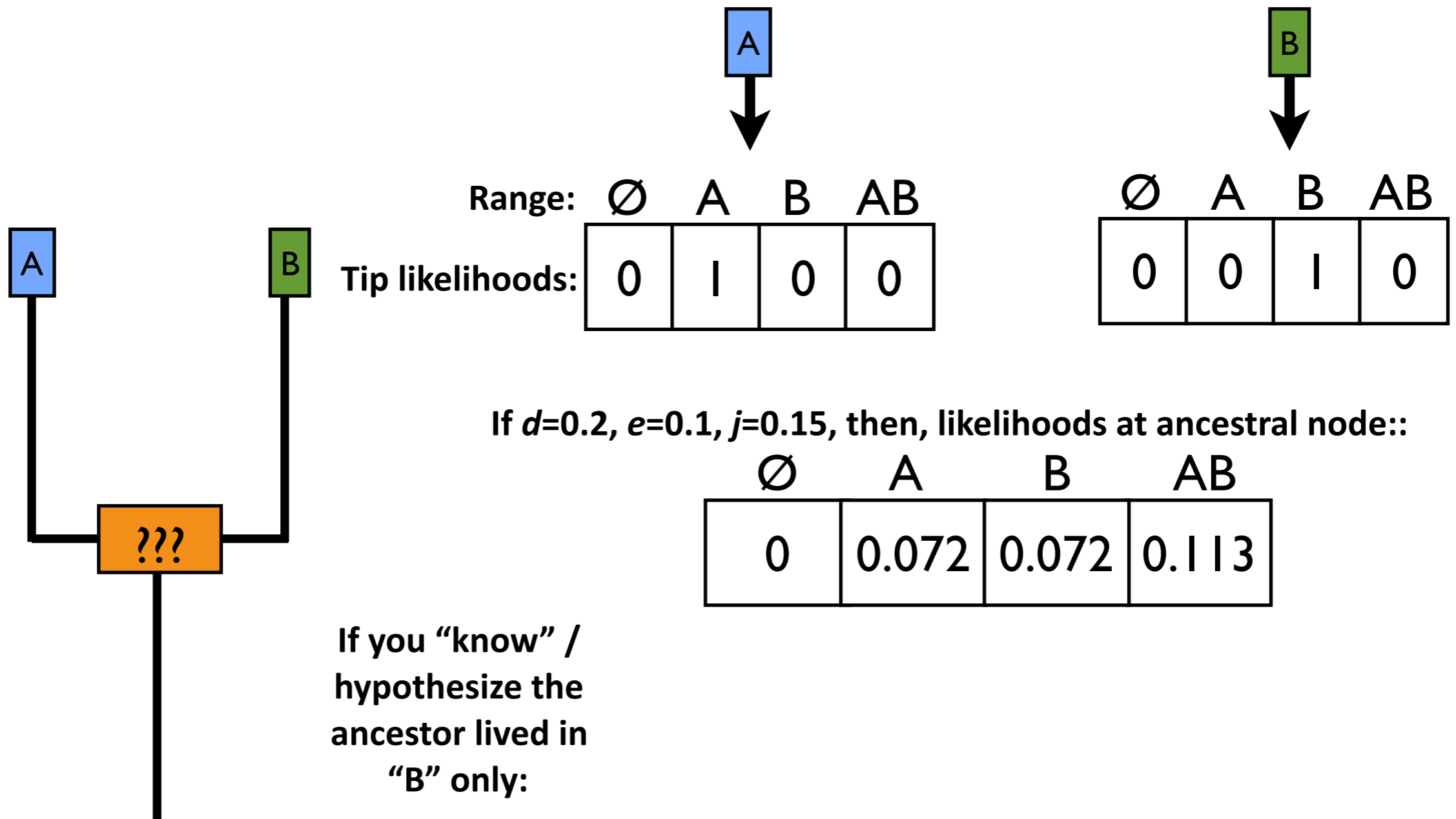
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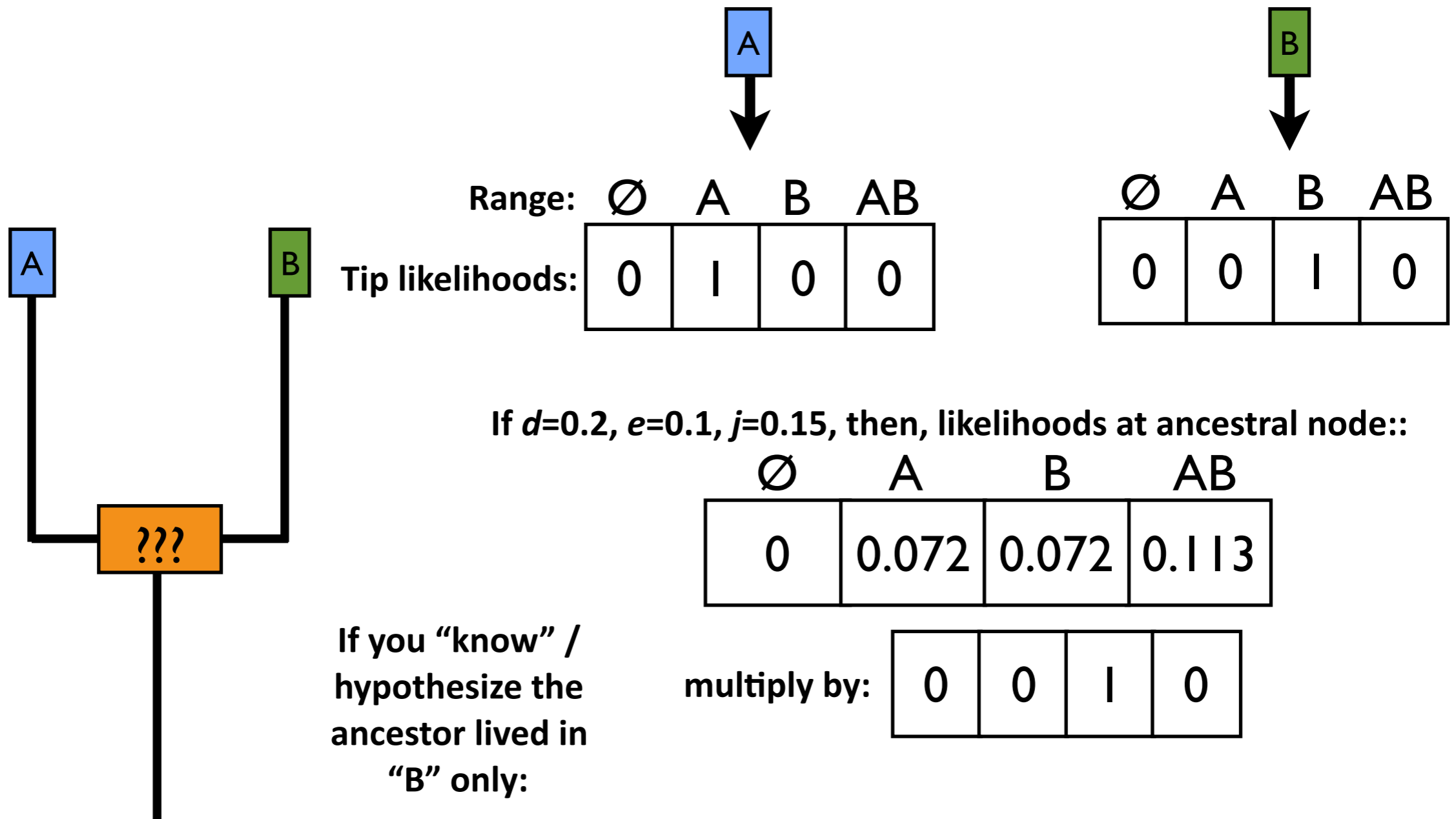
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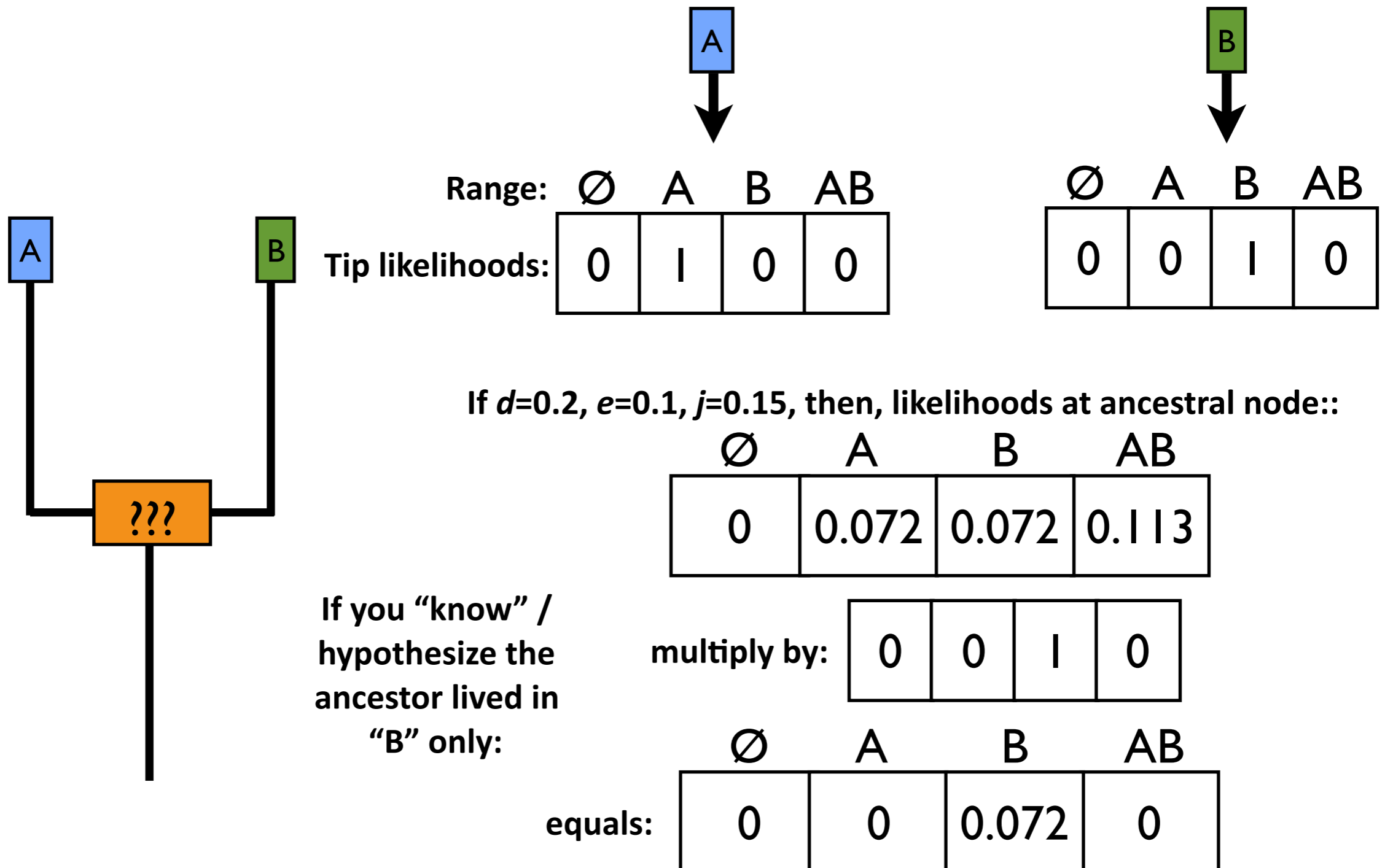
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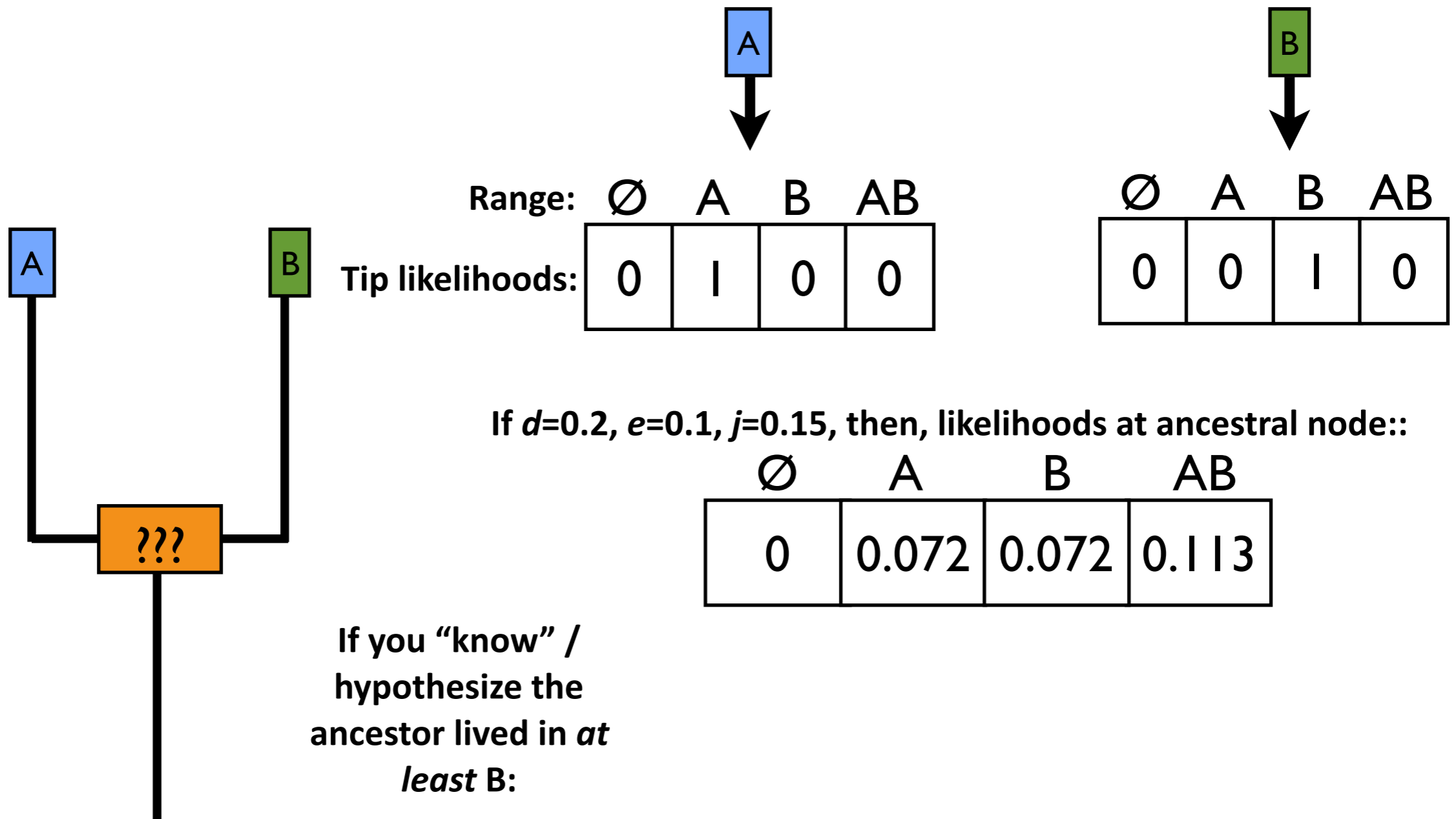
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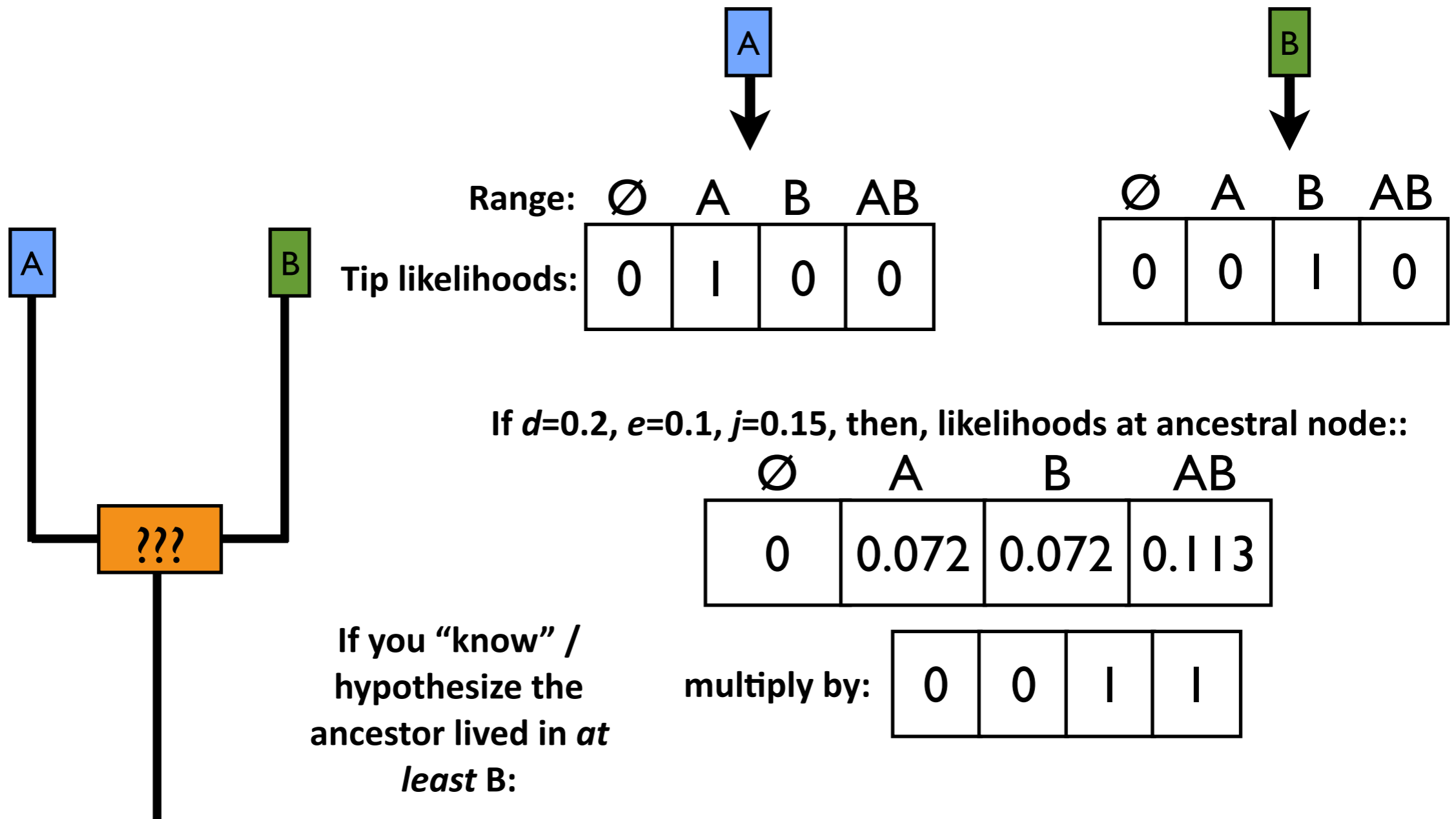
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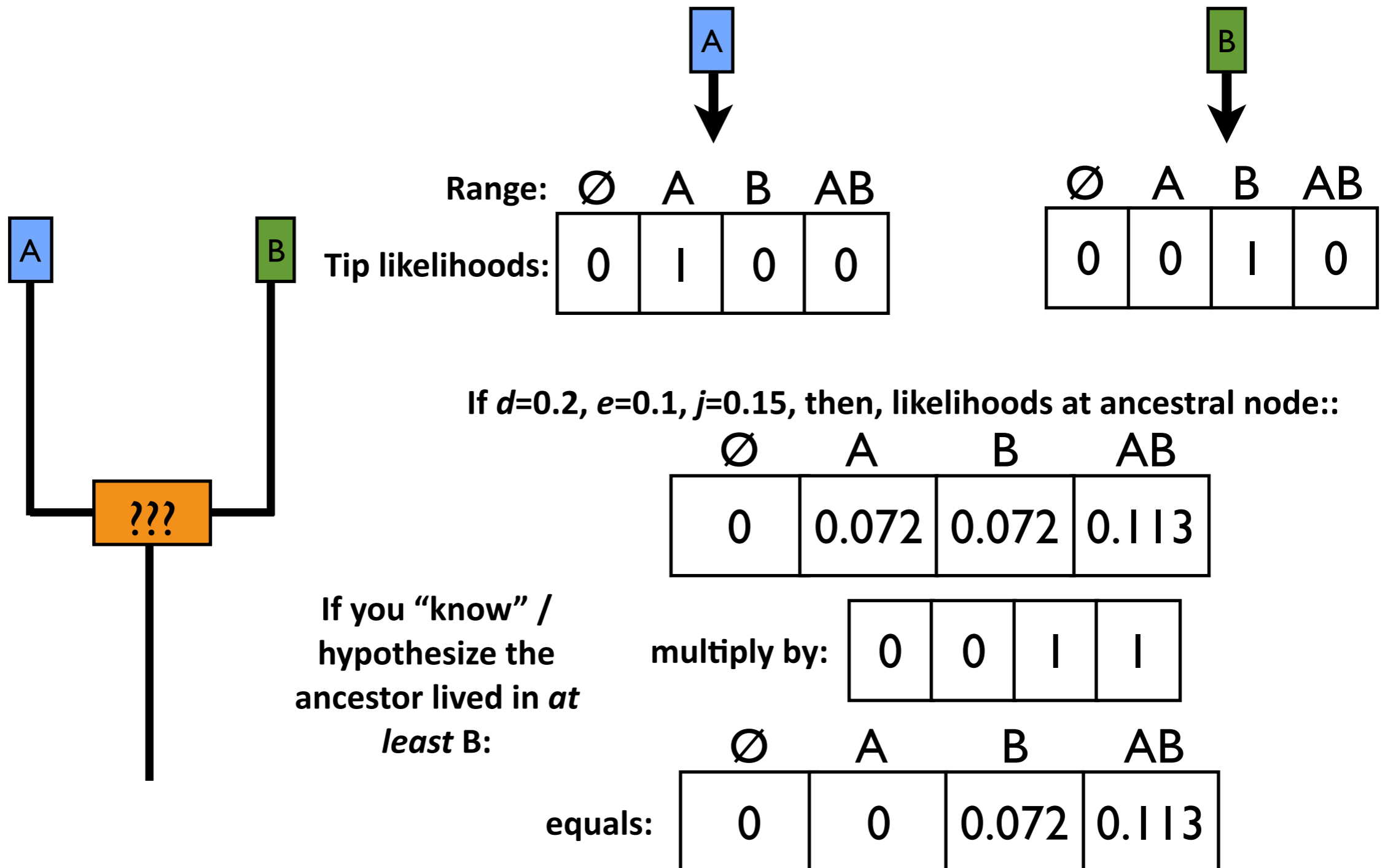
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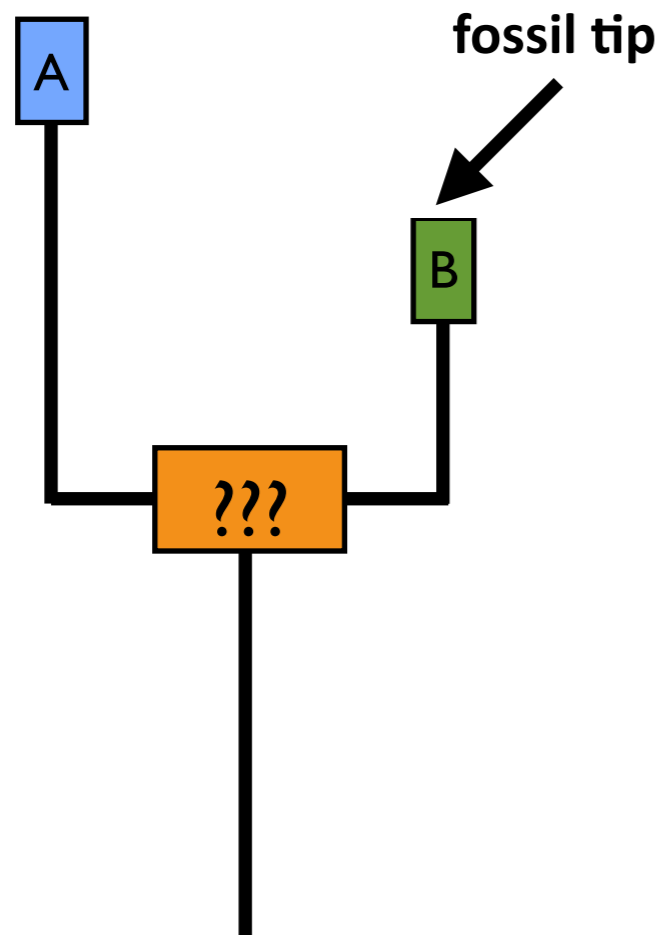
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Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

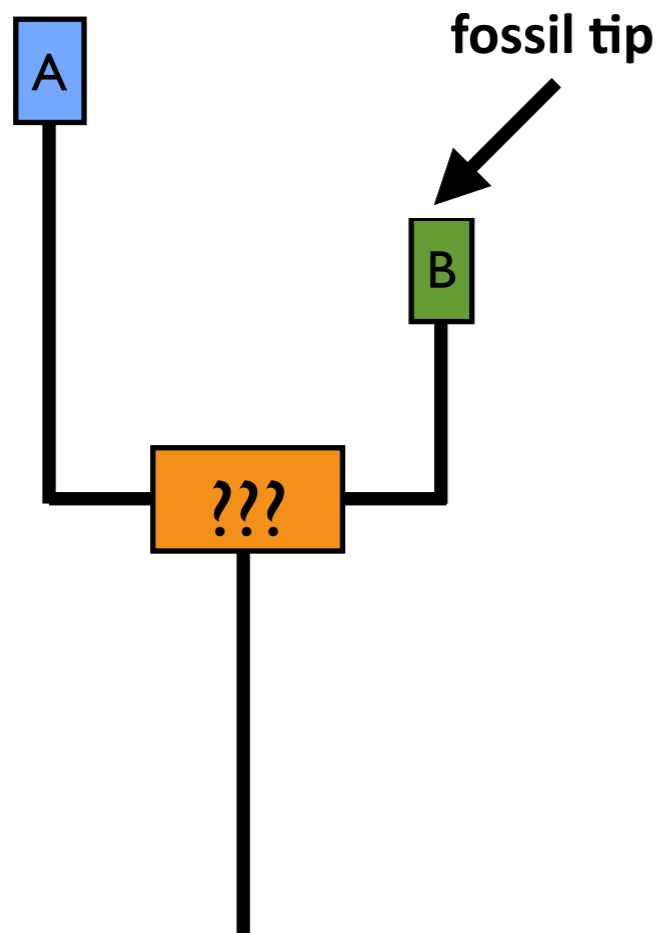
Strategy #2. Including fossils as terminal taxa



Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

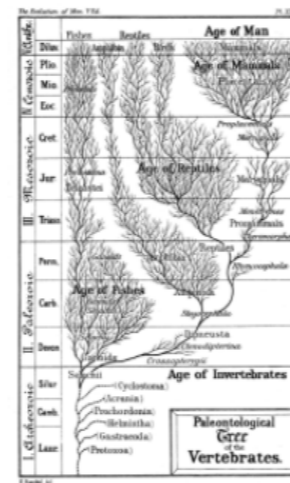
Strategy #2. Including fossils as terminal taxa

See *Biology Letters* Special Issue, “Putting Fossils in Trees,” with David Bapst, Graeme Lloyd, April Wright



THE ROYAL SOCIETY

Putting fossils in trees: combining morphology, time, and molecules to estimate phylogenies and divergence times



Articles from across the tree of life as well as issues concerning data quality and methodological choices have been compiled as part of this Special Feature. The papers included span a breadth of topics relating to placing fossils in trees and using them in time-scaling but with particular emphasis on, or comparison to, the suite of new "tip-dating" approaches.

Keywords: Evolution, Palaeontology, Taxonomy and systematics, Bioinformatics

Showing 1 - 9 of 9 items in this collection

Editor's note on 'Putting fossils in trees' special issue

Journal: *Biology Letters*

Date: Mar 22 2017

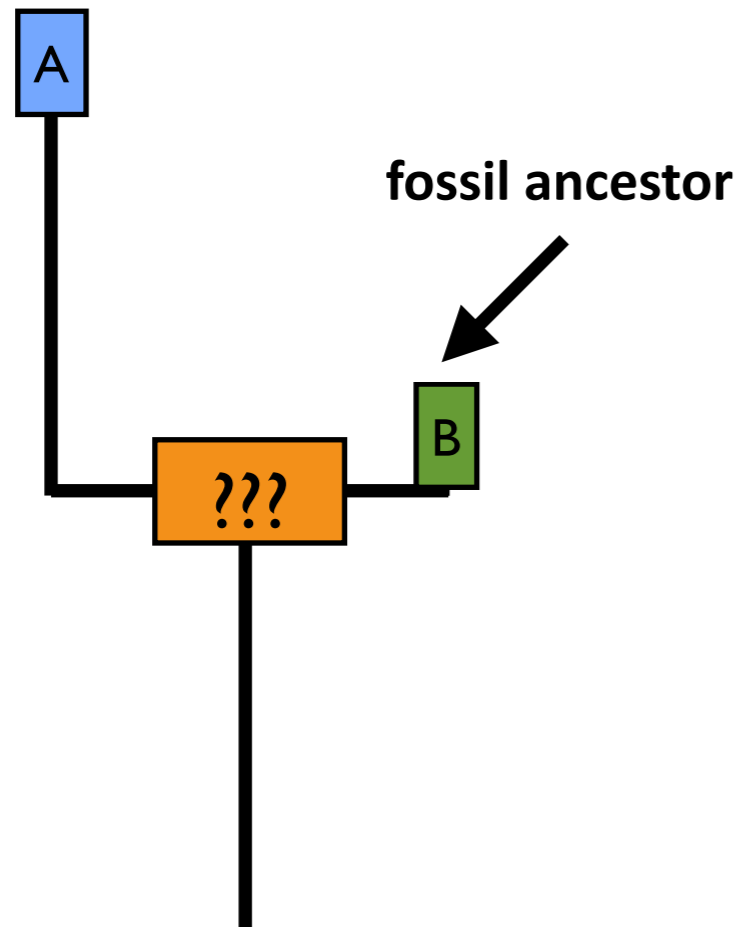
Time-calibrated models support congruency between Cretaceous continental rifting and titanosaurian evolutionary history

Journal: *Biology Letters*

Date: Apr 05 2016

Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

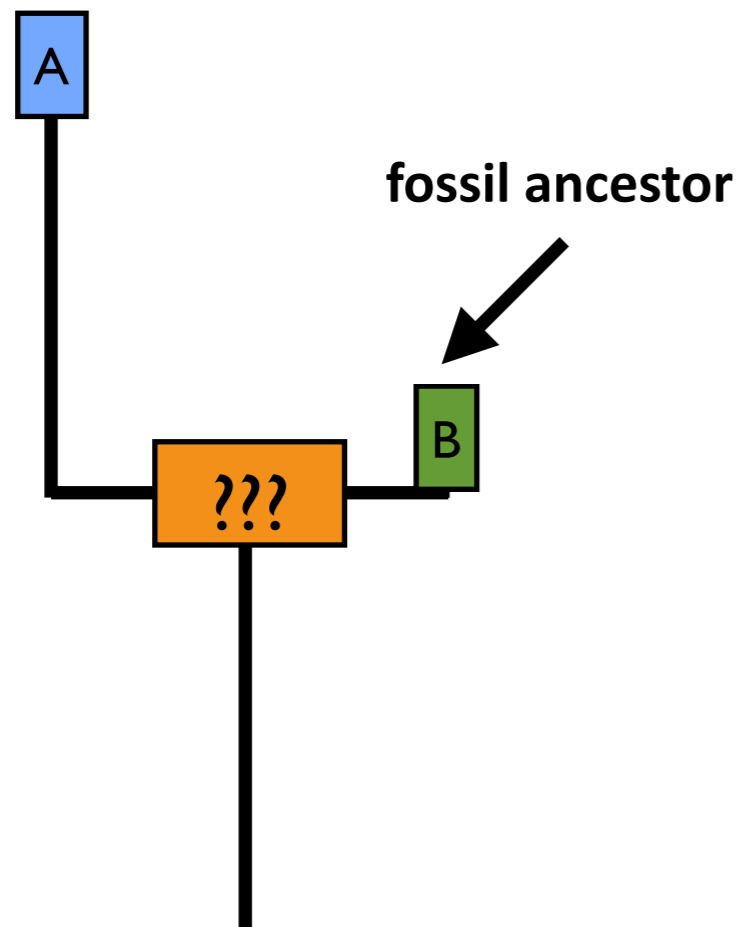
Strategy #3. Including fossils as direct ancestors



Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Strategy #3. Including fossils as direct ancestors

In *BioGeoBEARS*:

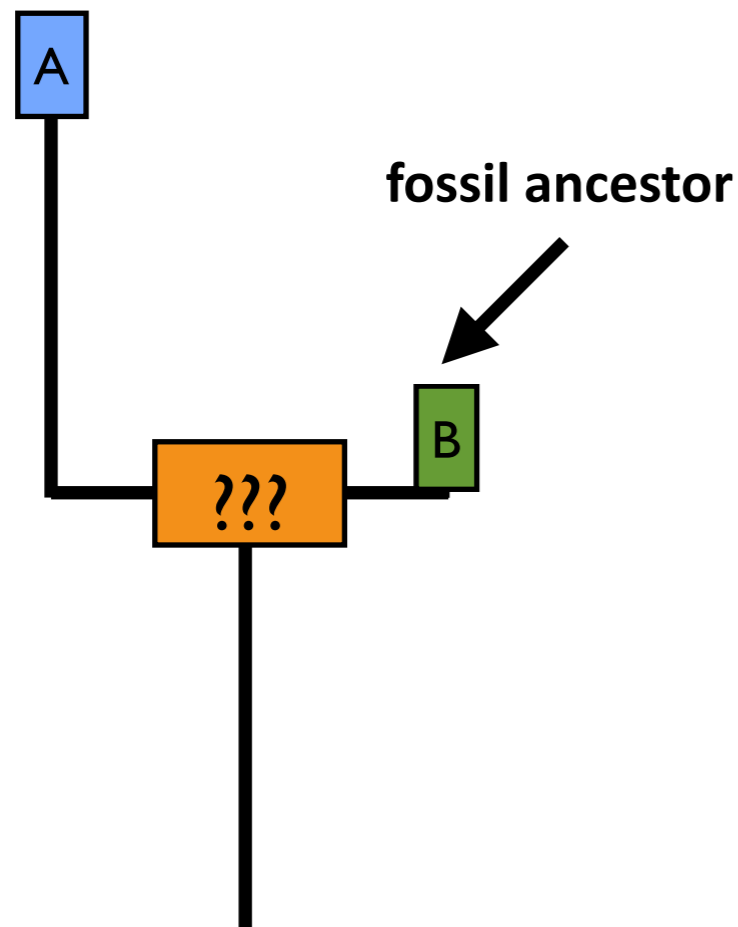


Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Strategy #3. Including fossils as direct ancestors

In *BioGeoBEARS*:

- fossil ancestors are identified as “hooks”

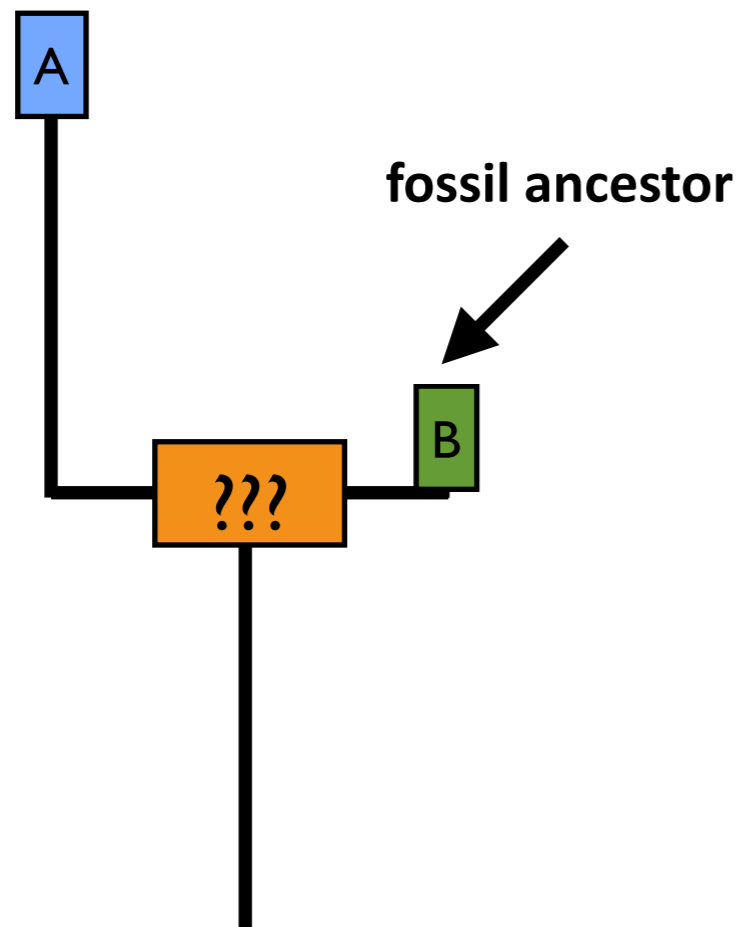


Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Strategy #3. Including fossils as direct ancestors

In *BioGeoBEARS*:

- fossil ancestors are identified as “hooks”
- “hook” = branch lengths < 0.000001 (or, user-specified)

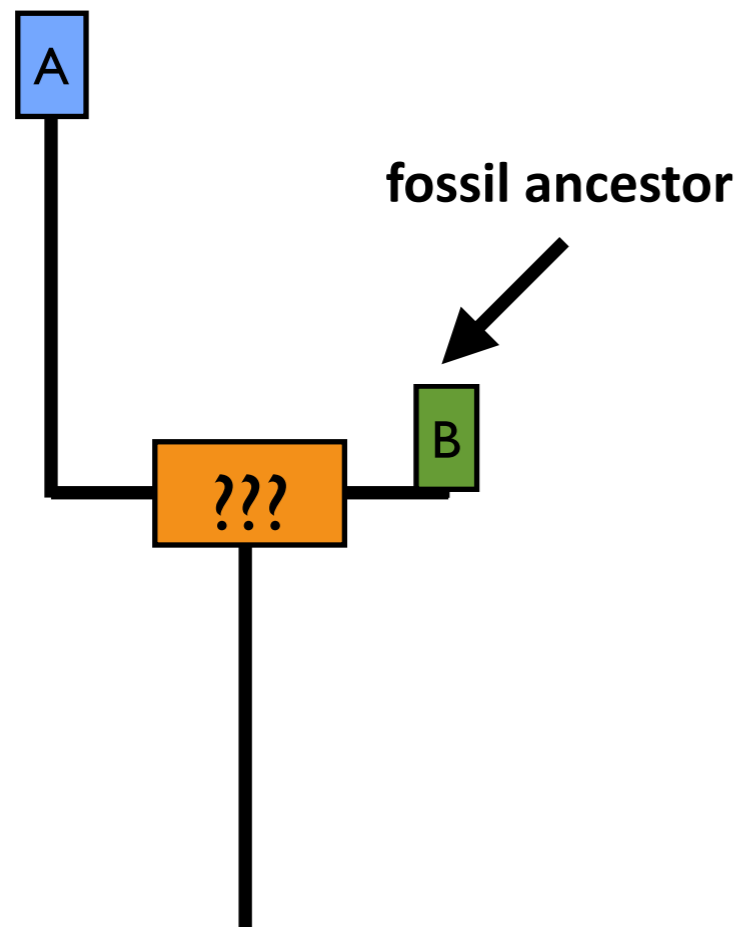


Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Strategy #3. Including fossils as direct ancestors

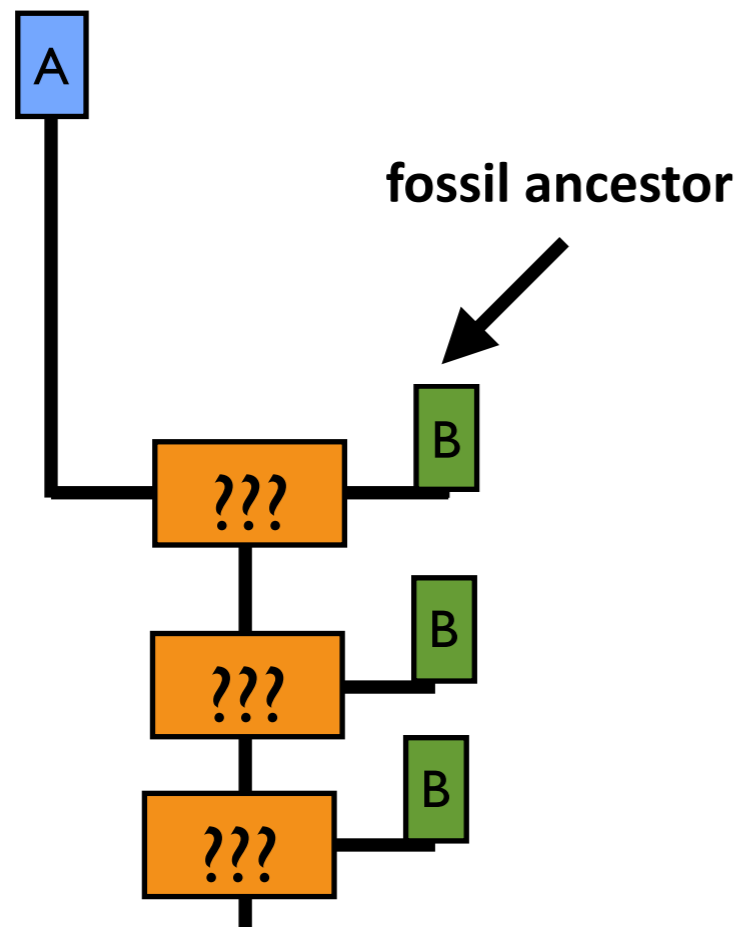
In *BioGeoBEARS*:

- fossil ancestors are identified as “hooks”
- “hook” = branch lengths < 0.000001 (or, user-specified)
- no cladogenesis process applied as these nodes are not real speciation events



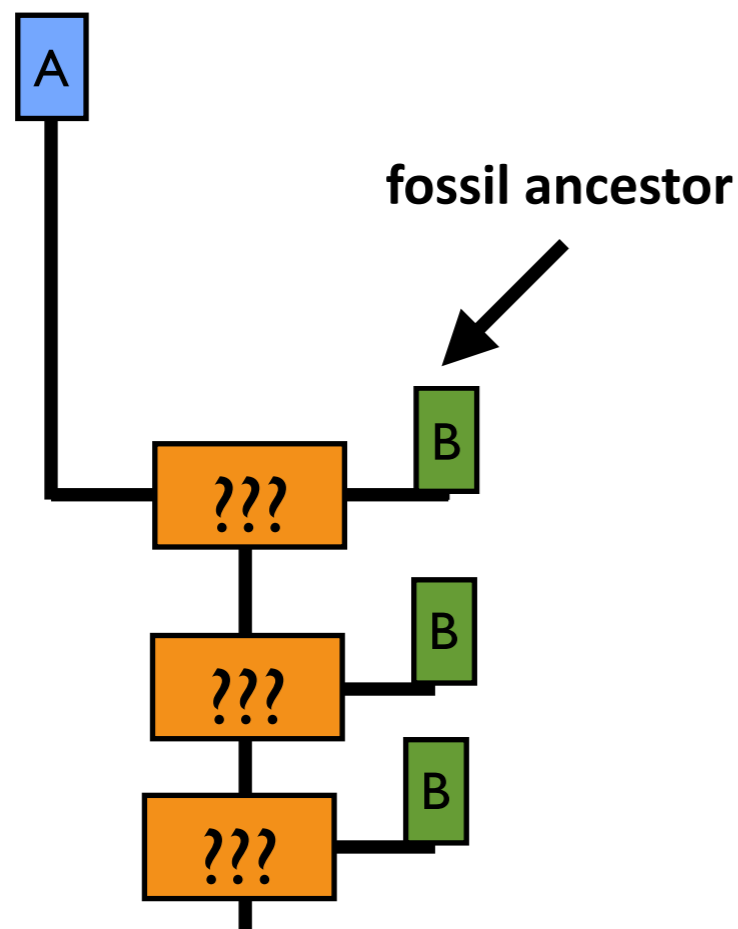
Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Strategy #4. Fossil species with time-ranges?



Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Strategy #4. Fossil species with time-ranges?

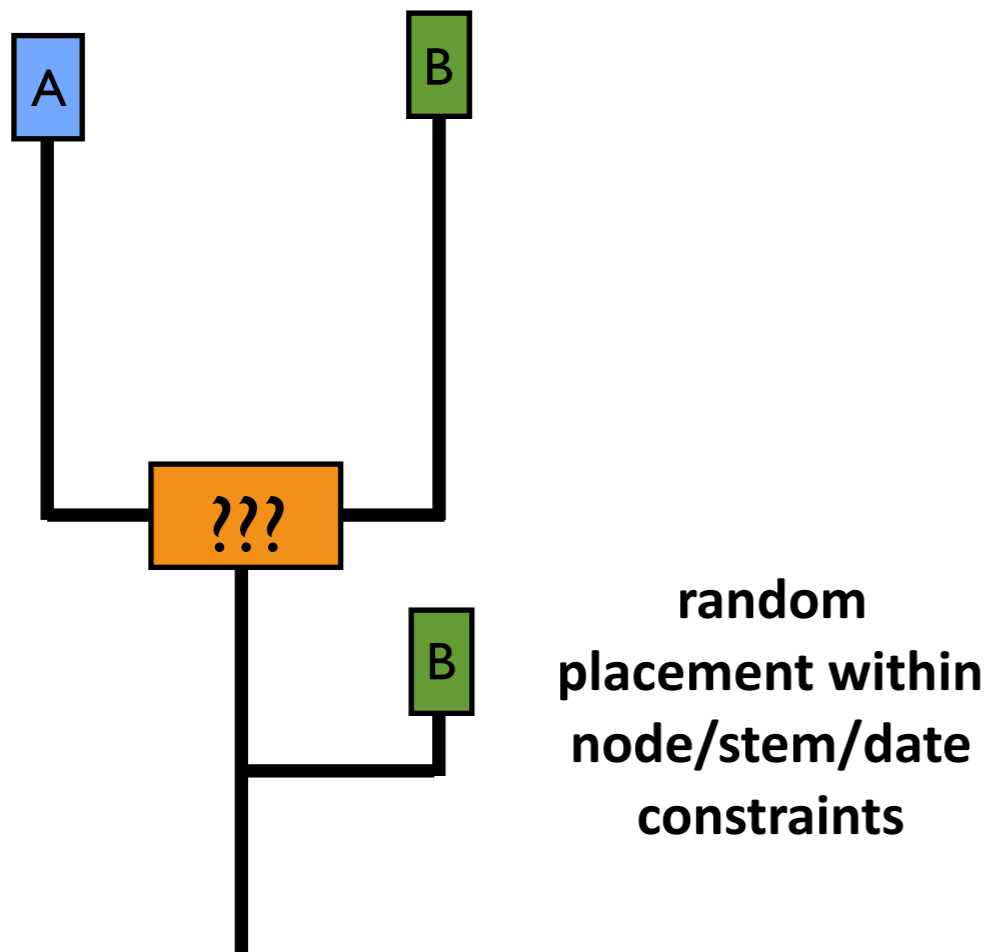


In *BioGeoBEARS*:

- add “hooks” at the midpoint of each geologic unit
- (or a timepoint selected from a uniform prior)

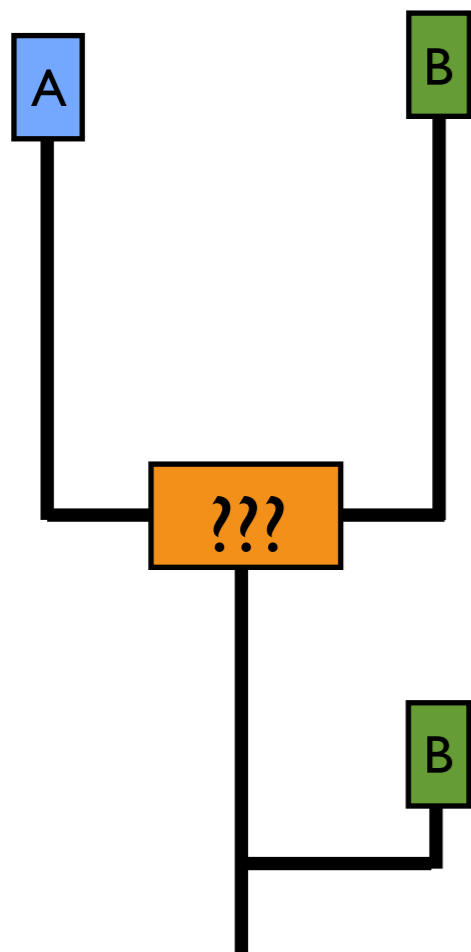
Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Strategy #5. What about fossils without characters?



Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

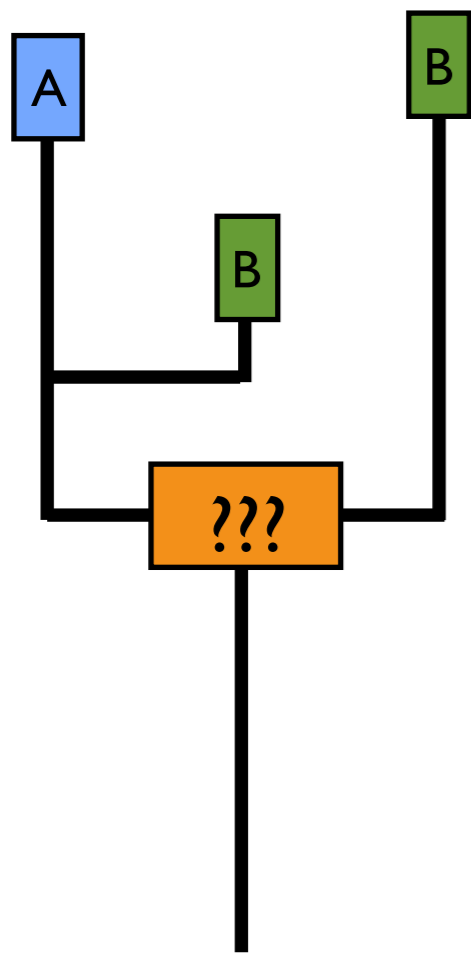
Strategy #5. What about fossils without characters?



random
placement within
node/stem/date
constraints

Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

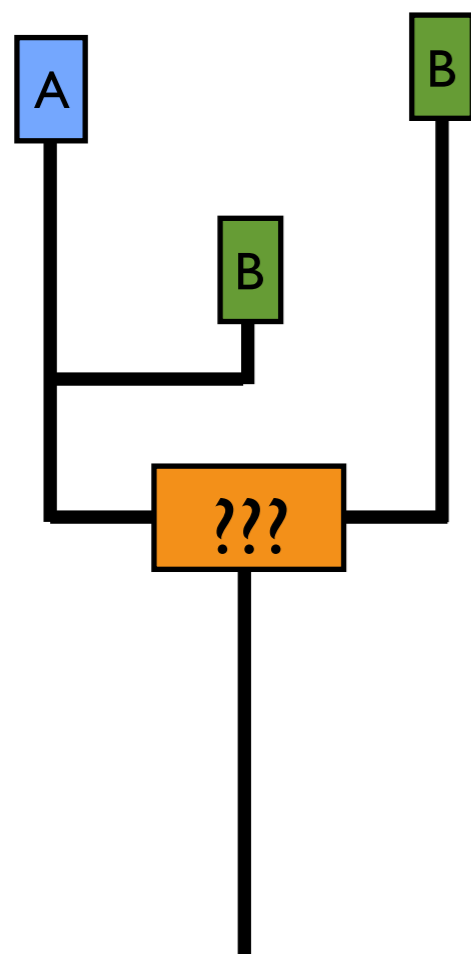
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Strategy #5. What about fossils without characters?

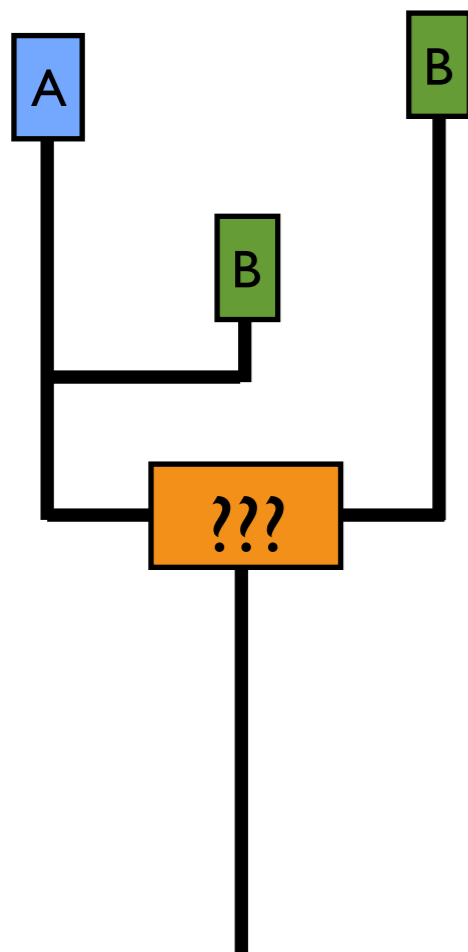


random
placement within
node/stem/date
constraints

- Repeat random placement e.g. 100 times, or randomly place on 100 posterior trees
- Not as desirable as placing fossils according to a birth/death/sampling prior in BEAST, but more flexible

Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Big issue: are fossil ranges equal to real ranges?

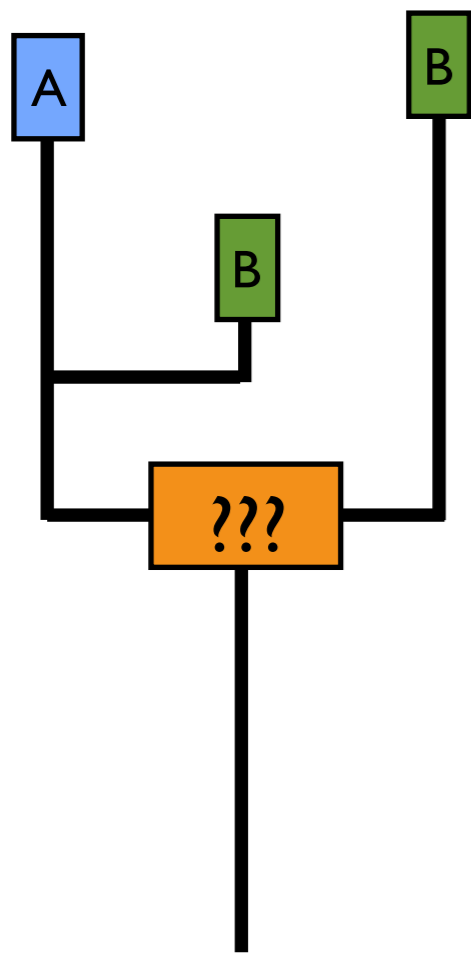


If a fossil is found in area “B”

- sometimes, you can assume “B” is the complete range (maybe)
- But perhaps the real range was AB
- (In some cases, this could even be an issue with living taxa)

Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

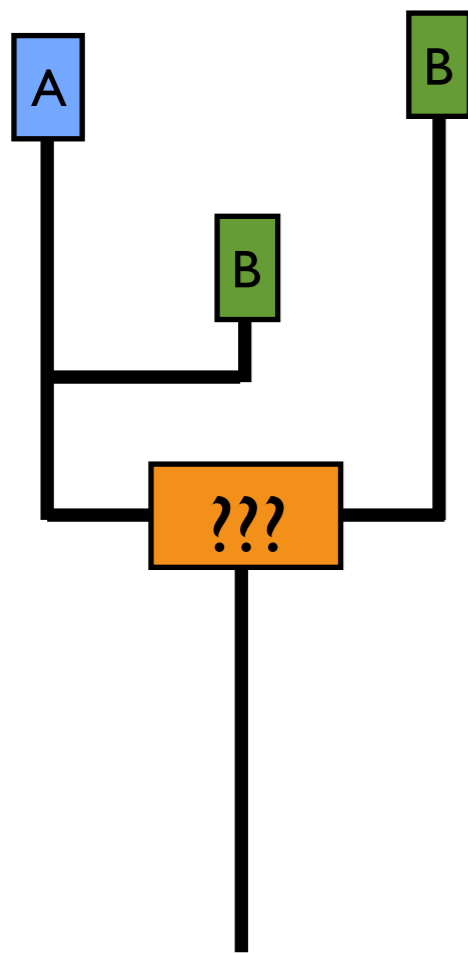
Big issue: are fossil ranges equal to real ranges?



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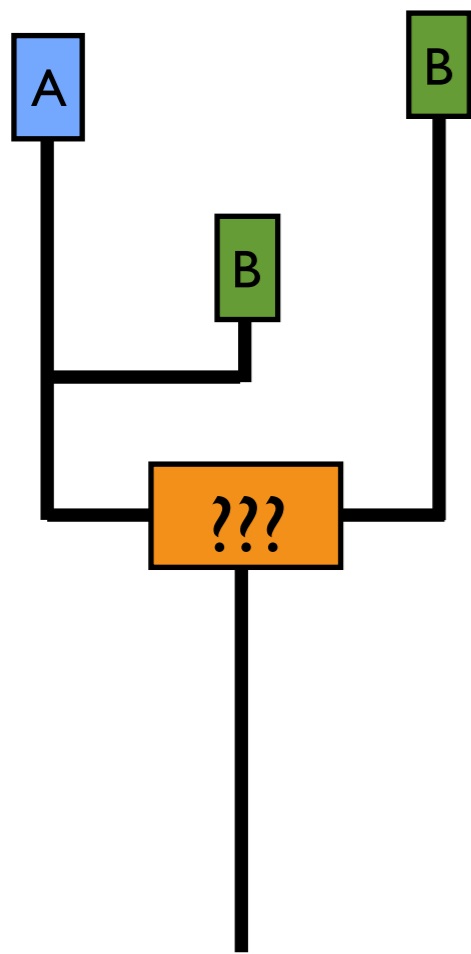
Big issue: are fossil ranges equal to real ranges?

Incomplete ranges strategy #1:



Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Big issue: are fossil ranges equal to real ranges?



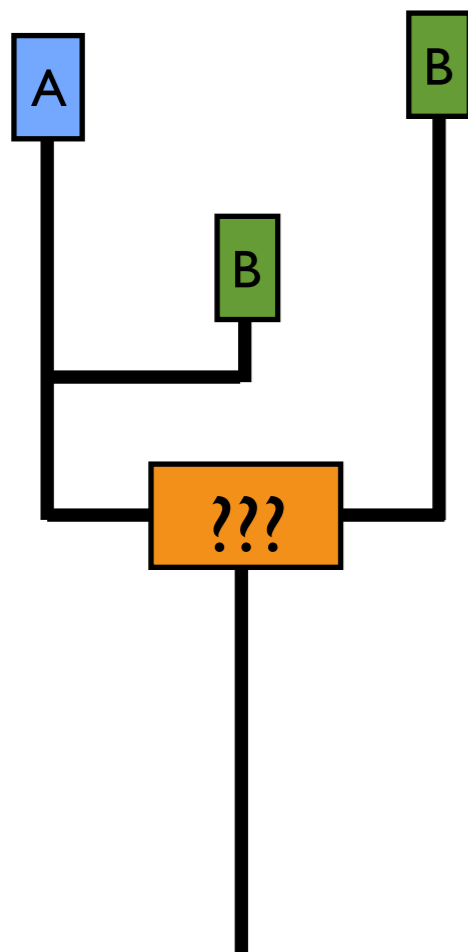
Incomplete ranges strategy #1:

Code with
question
marks:

| | |
|---|---|
| A | B |
| ? | I |

Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Big issue: are fossil ranges equal to real ranges?



Incomplete ranges strategy #1:

Code with
question
marks:

| | |
|---|---|
| A | B |
| ? | I |

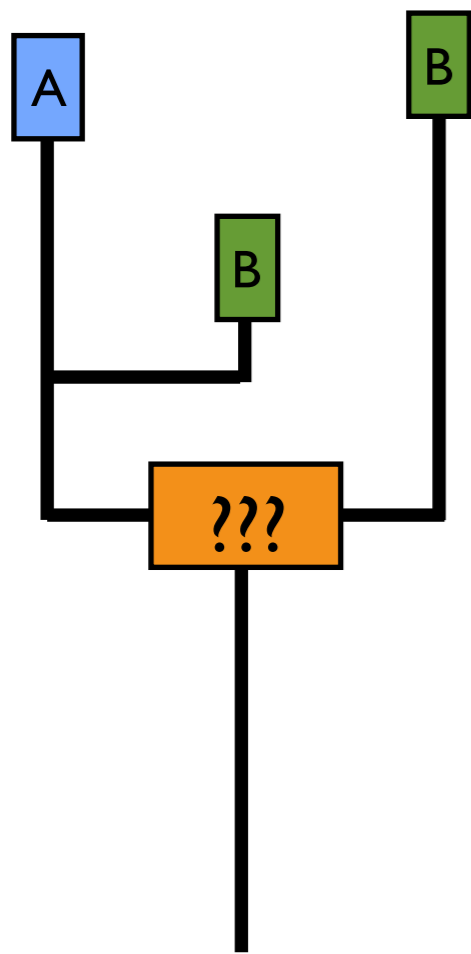
Tip likelihoods:

| \emptyset | A | B | AB |
|-------------|---|---|----|
| 0 | 0 | 1 | 1 |

**My experience: leads to very widespread
ancestors, weird parameters**

Putting fossils into model-based biogeography: strategies available in *BioGeoBEARS*

Big issue: are fossil ranges equal to real ranges?



Incomplete ranges strategy #2:

Have a *model* for imperfect detection

BioGeoBEARS model for imperfect detection

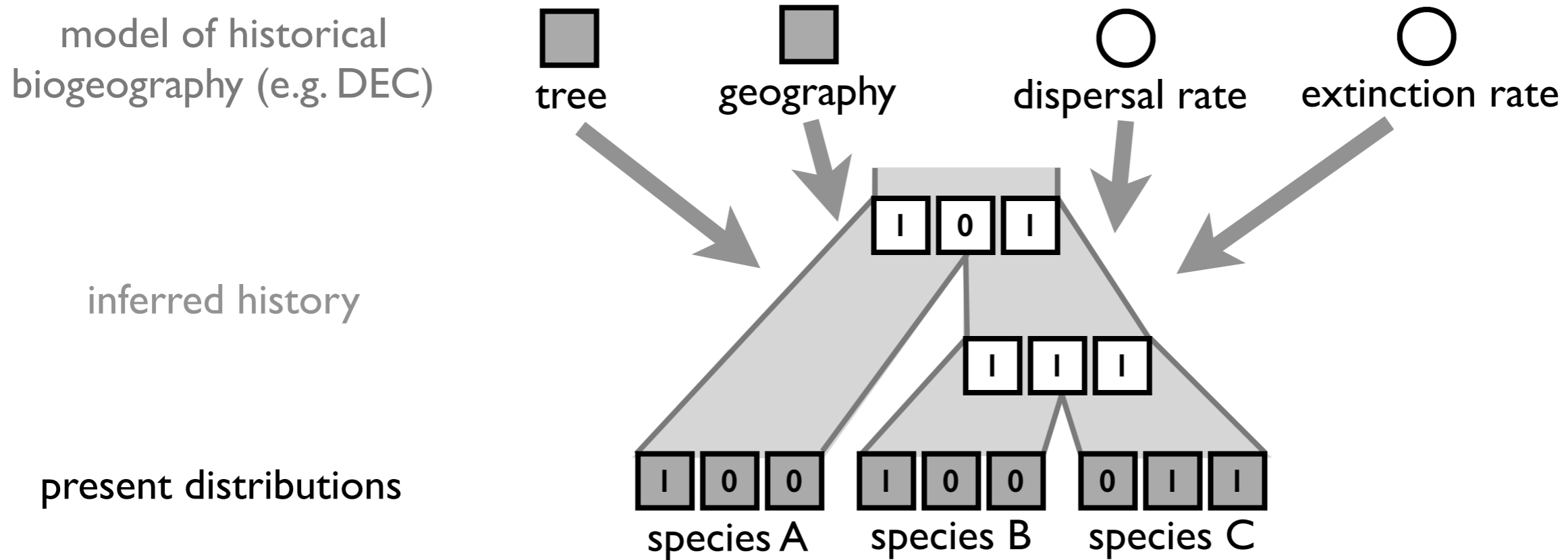
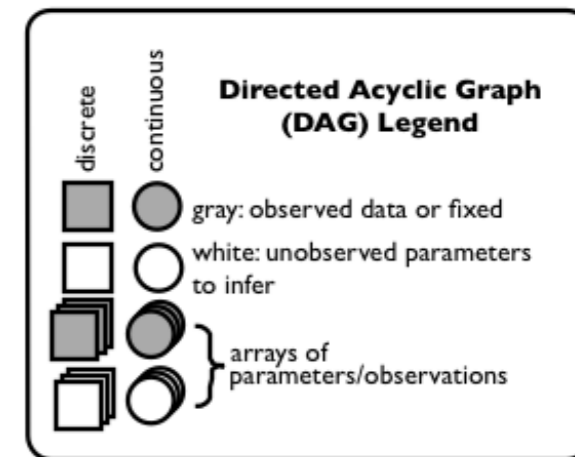


Figure 1A. Traditional inference procedure in likelihood analyses of historical biogeography on a phylogeny.

BioGeoBEARS model for imperfect detection

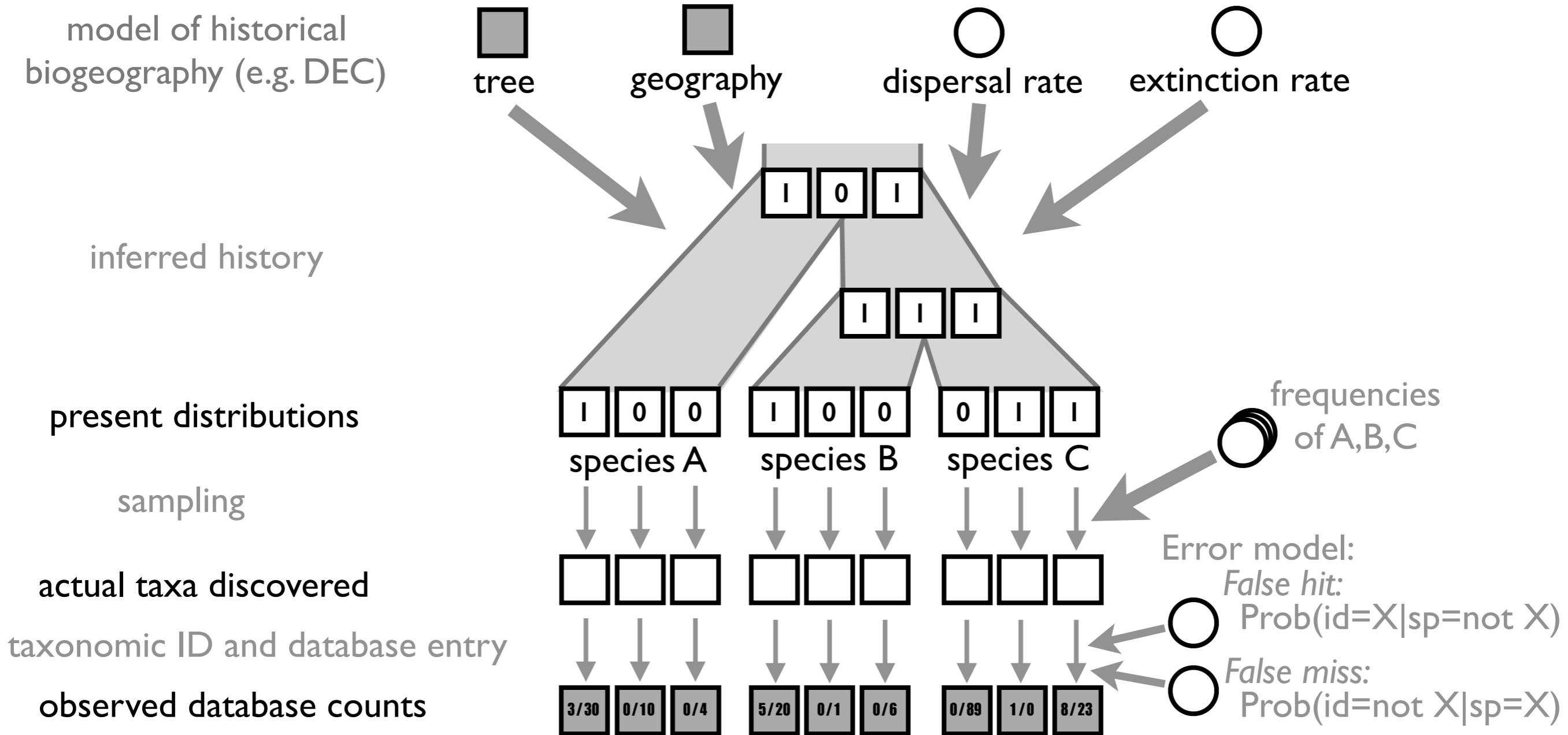
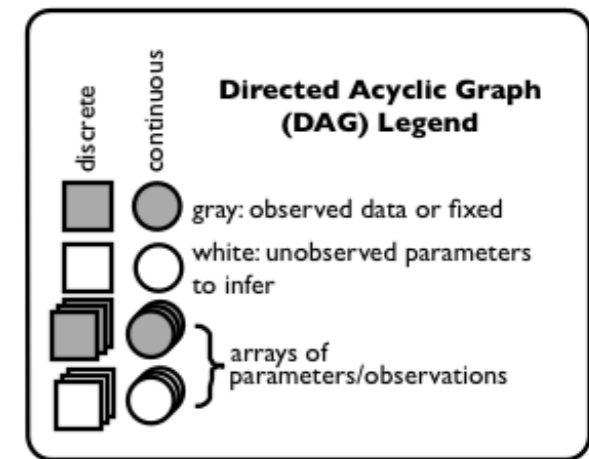


Figure 1B. DAG (Directed Acyclic Graph) for the model of imperfect detection implemented in BioGeoBEARS.

Imperfect detection: Let's try it!

Family Canidae

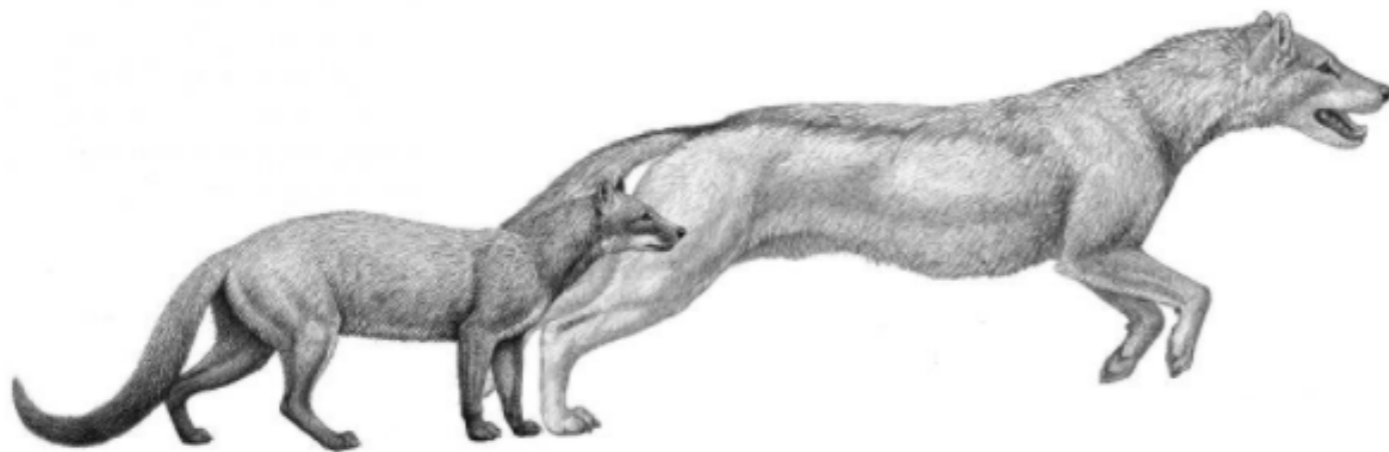
Fossilized Birth-Death analysis in *Beast2* (using *BEASTmasteR*)

~35 living species - DNA from Prevosti 2009, GenBank

**~120+ fossil species - morphology from Slater 2015,
Prevosti 2009**

**- “Taphonomic control”: background database of large
mammal occurrences: NEOMAP and NOW databases**

- Caninae occurrence database: assembled by Laura Säilä



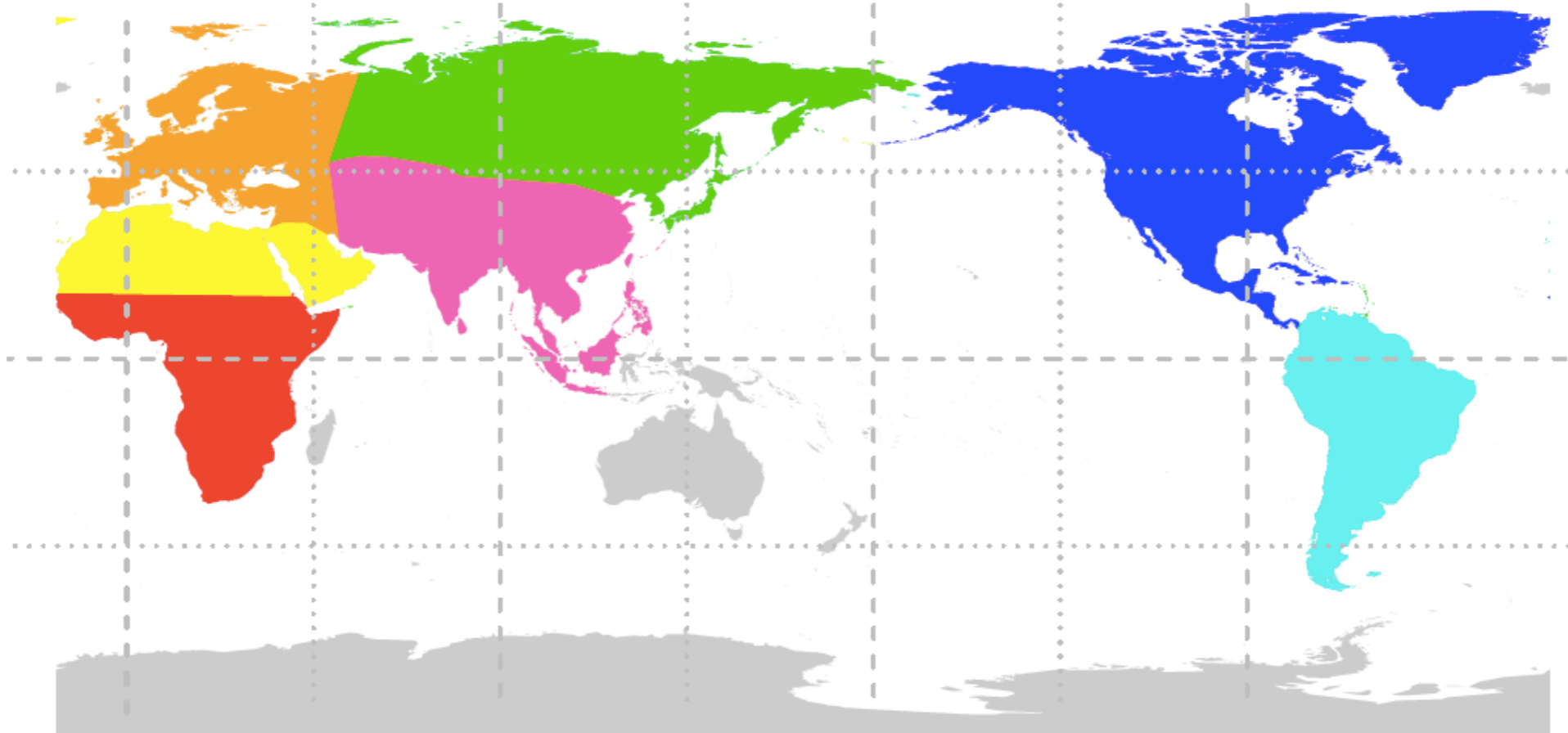
Hesperocyon and Sunkahetanka, two species of early dog. They were quite small and more closely resembled a mongoose.

(Illustration by Mauricio Anton)

Source: <http://www.forbes.com/sites/shaenamontanari/2015/08/18/old-dogs-learned-new-tricks-for-hunting-as-climate-changed/>

Imperfect detection: Let's try it!

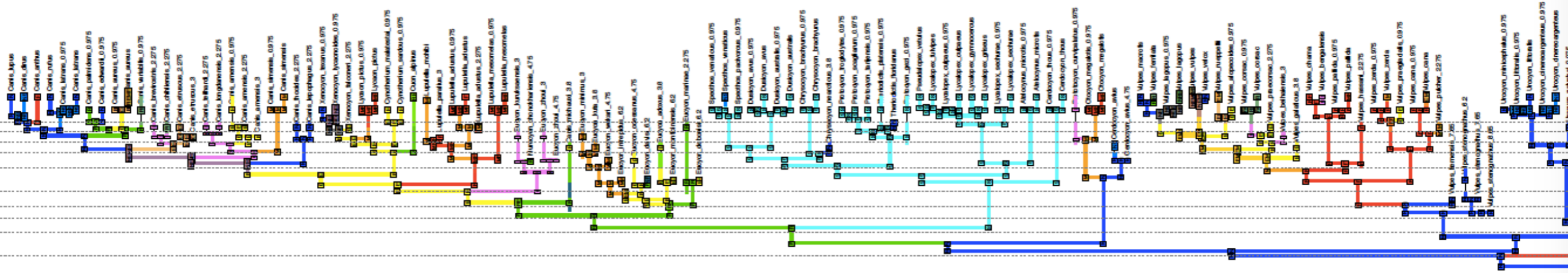
7 discrete regions:



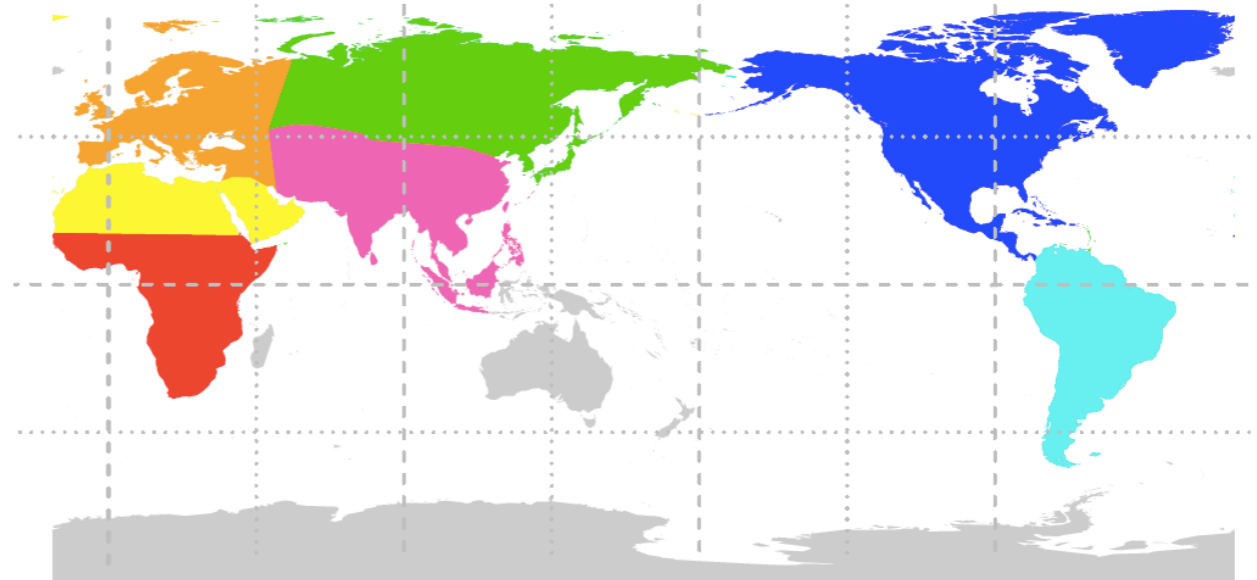
- Time-stratification: 11 time bins
- Panamanian isthmus, Beringia, Sahara Desert
- +w parameter to scale dispersal multipliers*
- Biogeographical Stochastic Mapping*

* = see Dupin, Matzke et al. 2016, *J. Biogeog*

Biogeographical Stochastic Map* #1

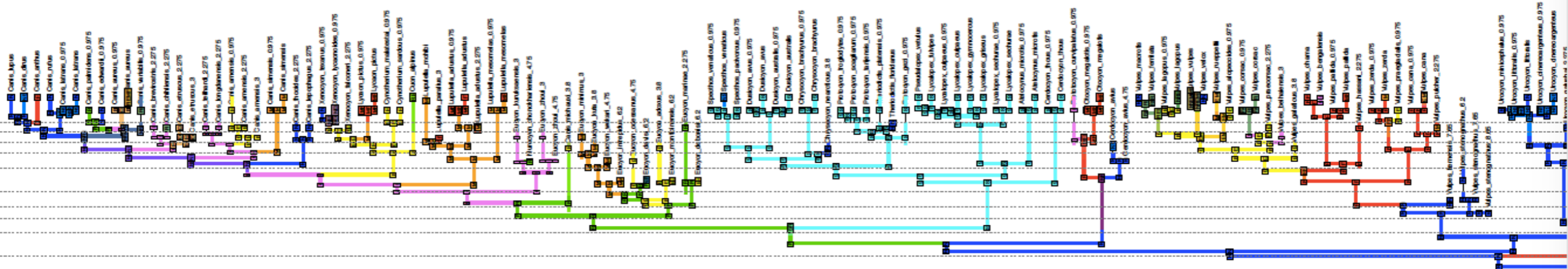


Area colors:

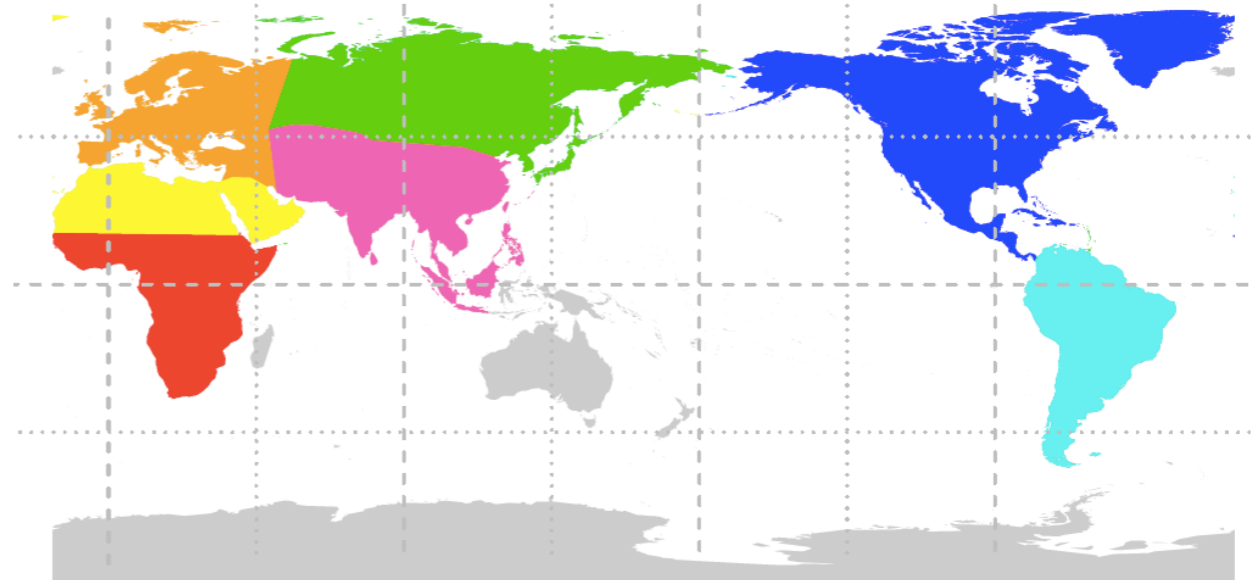


* = see Dupin, Matzke et al. 2016, *J. Biogeog*

Biogeographical Stochastic Map* #2

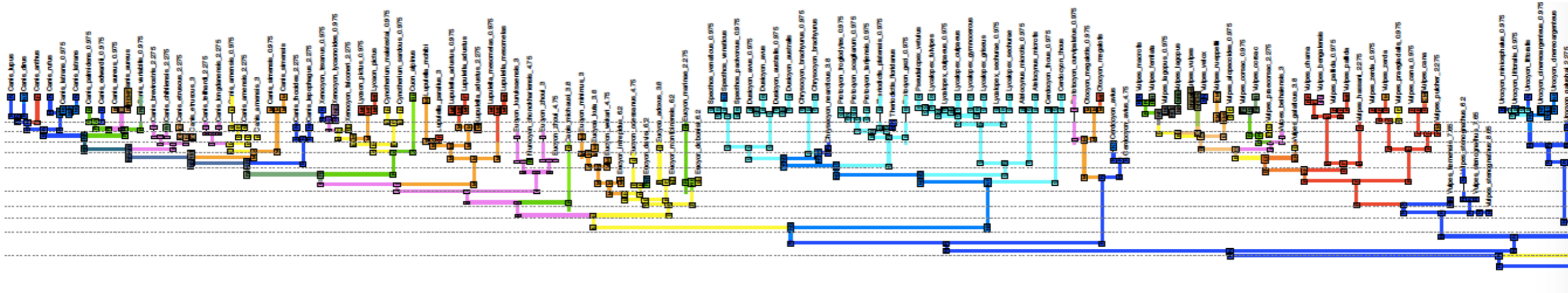


Area colors:

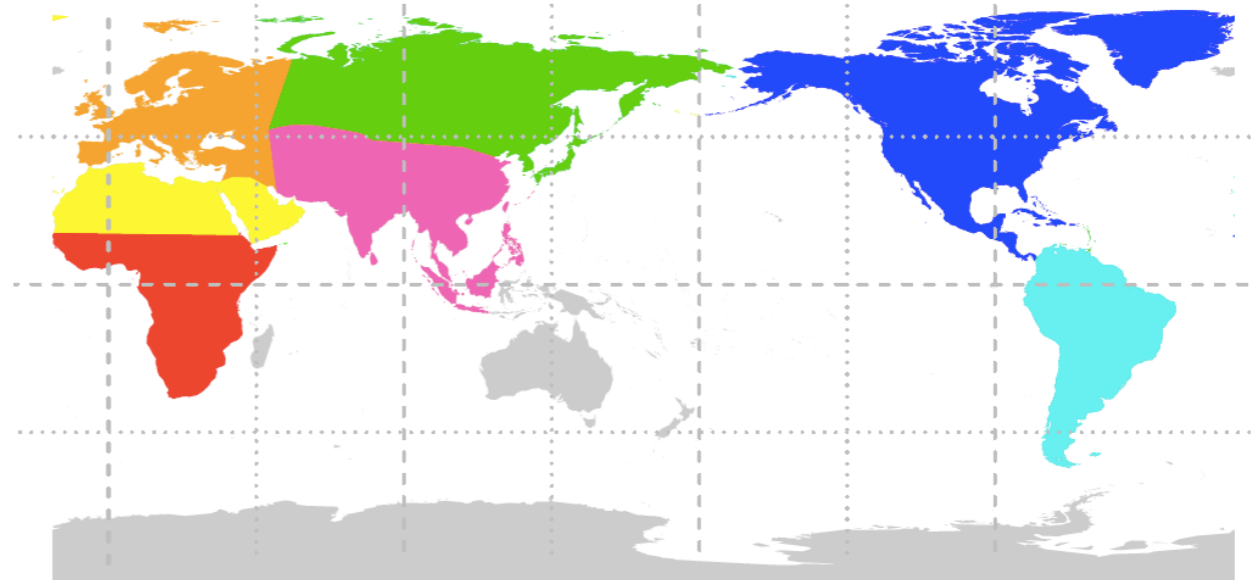


* = see Dupin, Matzke et al. 2016, *J. Biogeog*

Biogeographical Stochastic Map* #3

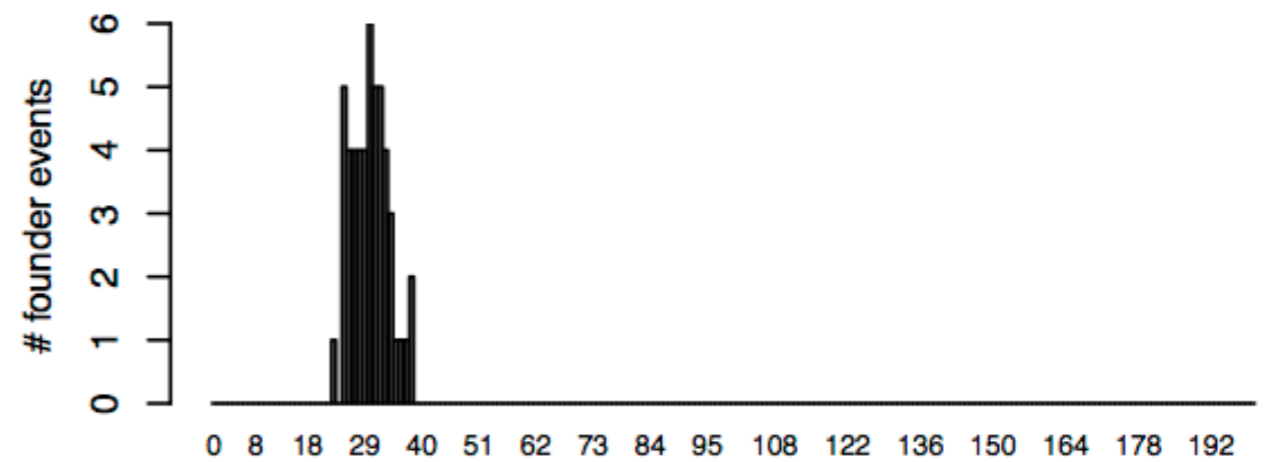
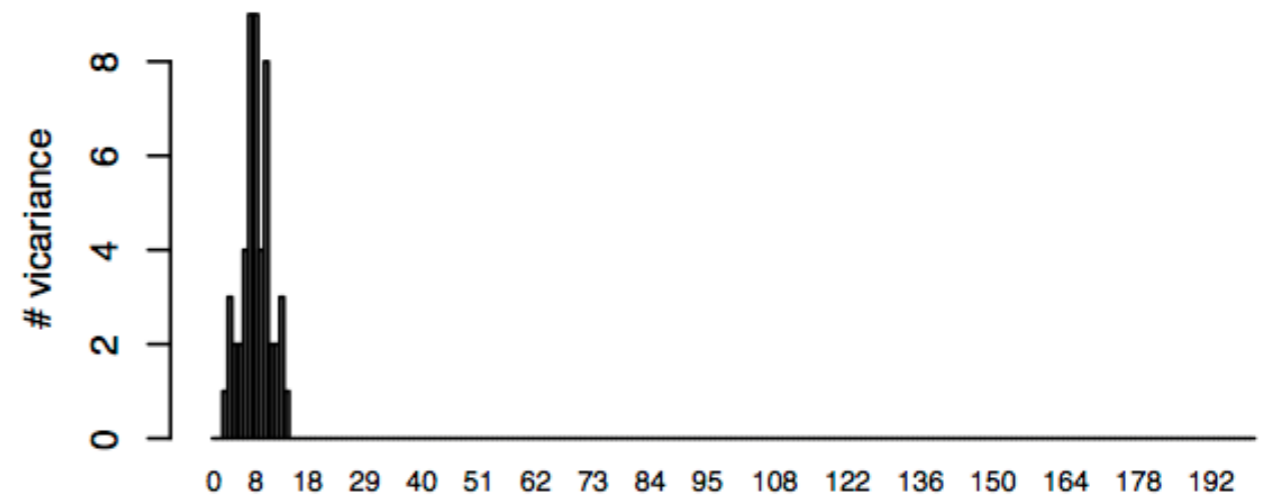
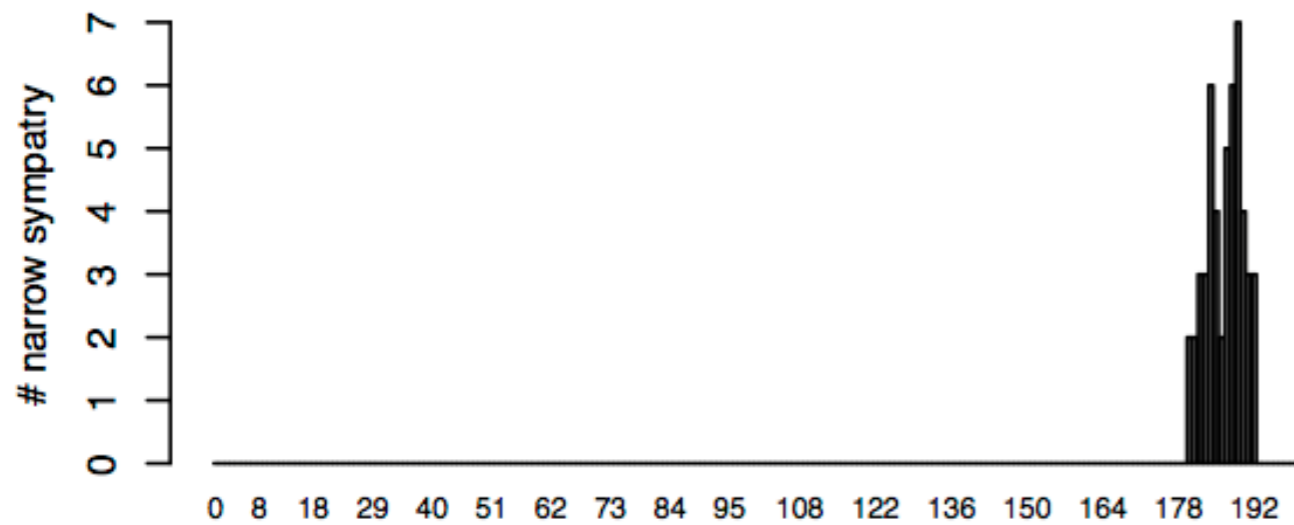
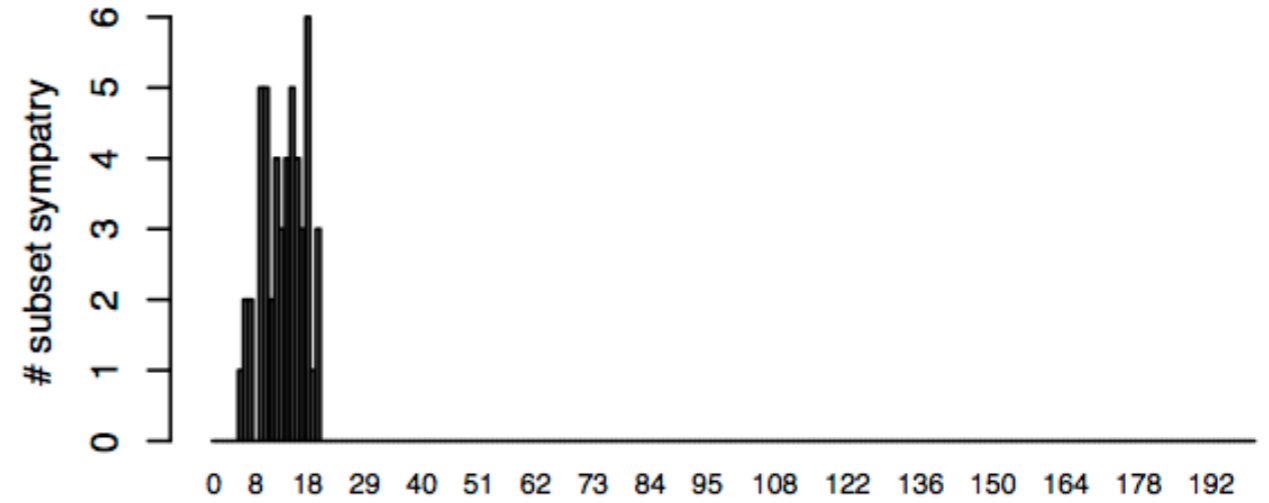
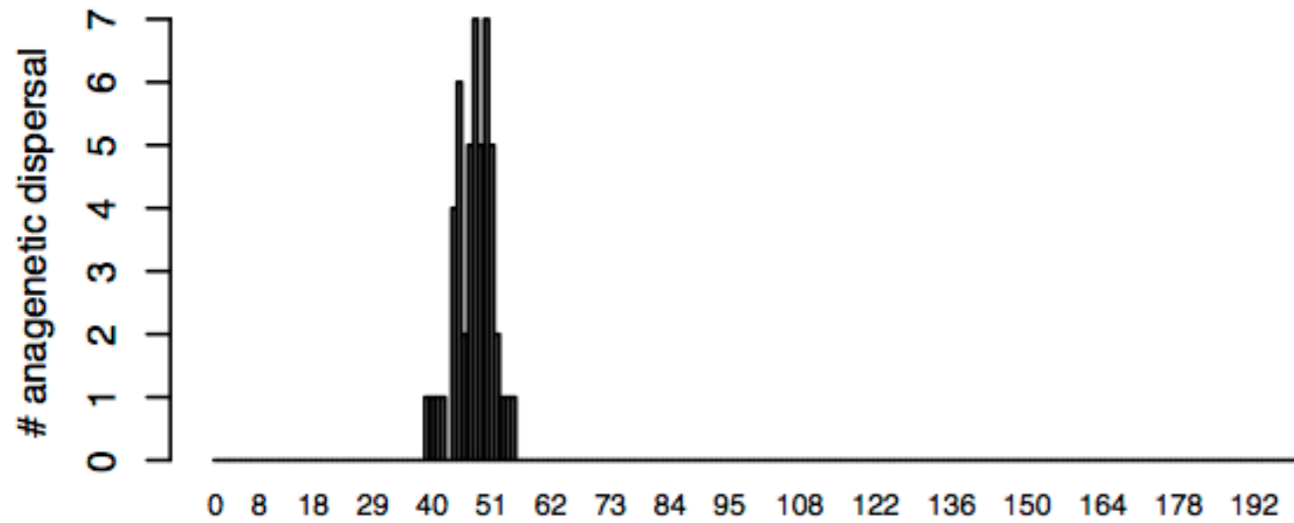


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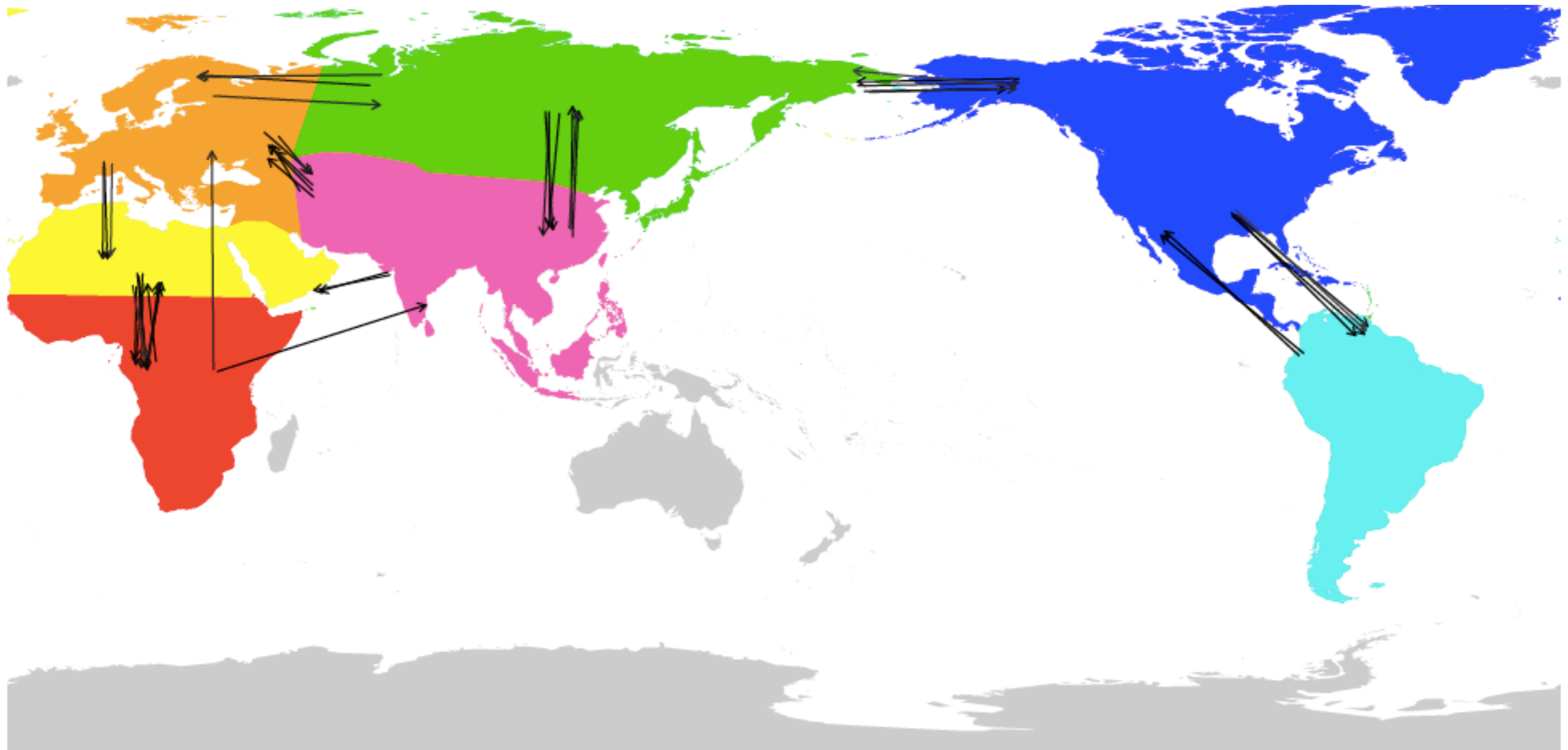
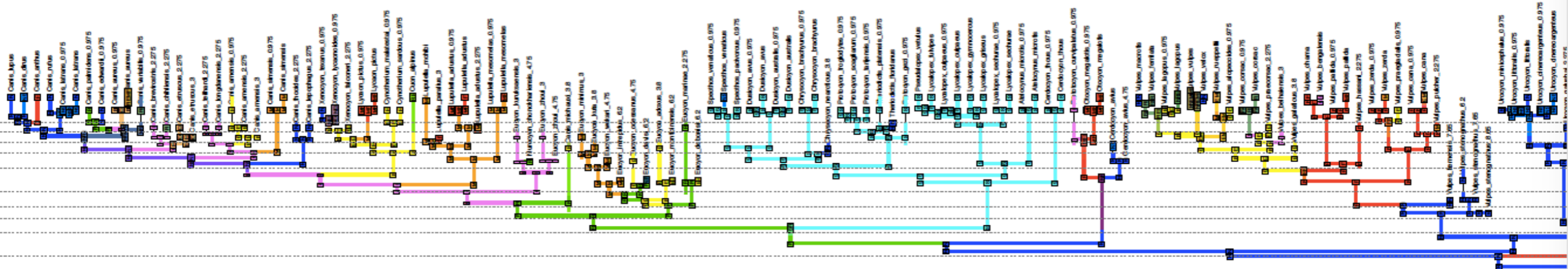
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Biogeographical Stochastic Maps: Event counts

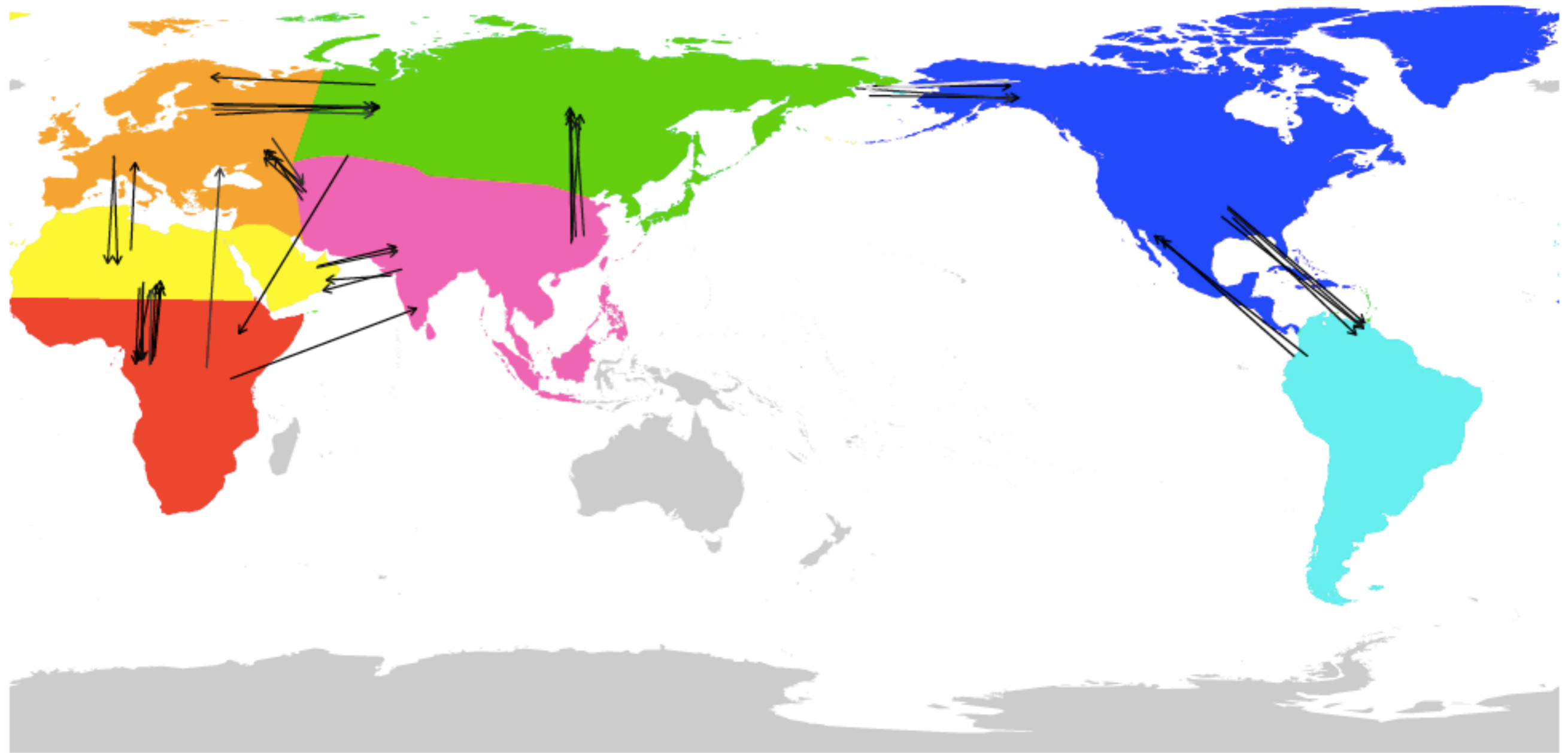
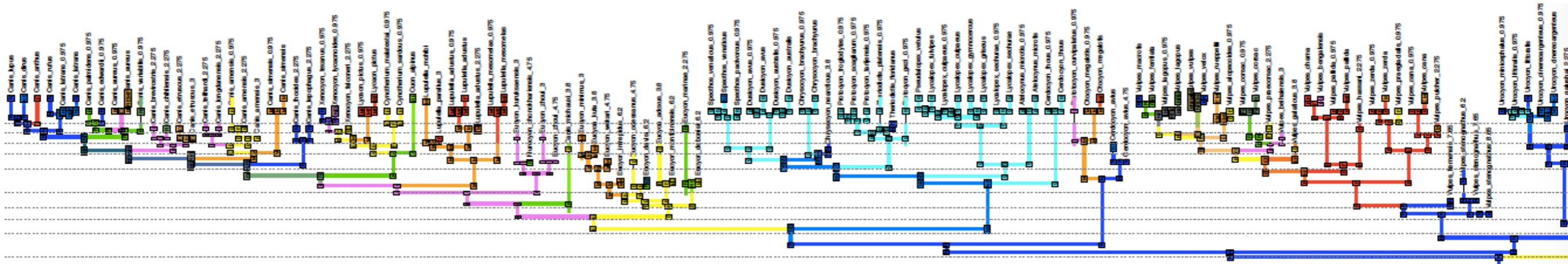


Event counts in each of 50 BSMs

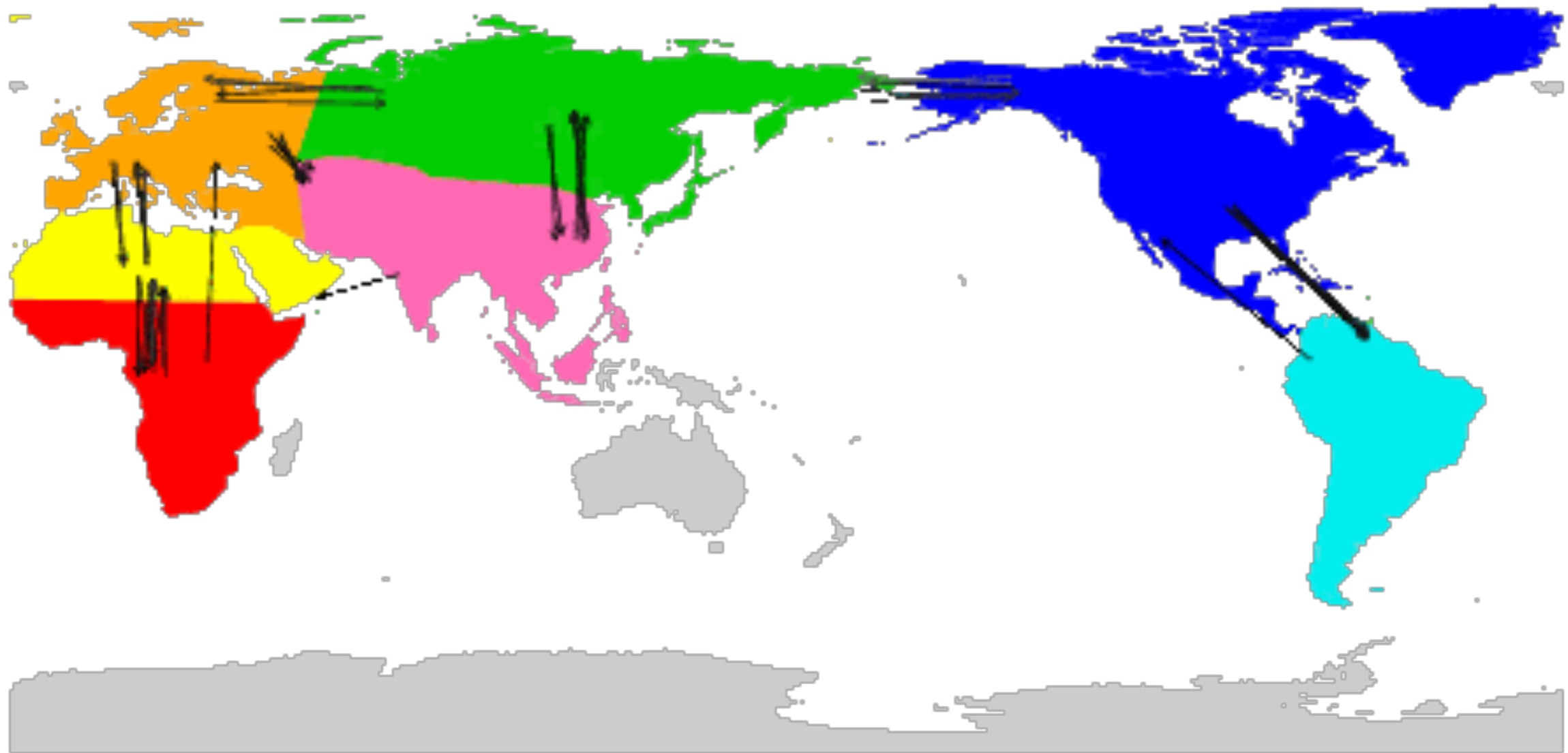
Mapping the stochastic maps



Mapping the stochastic maps



Animation



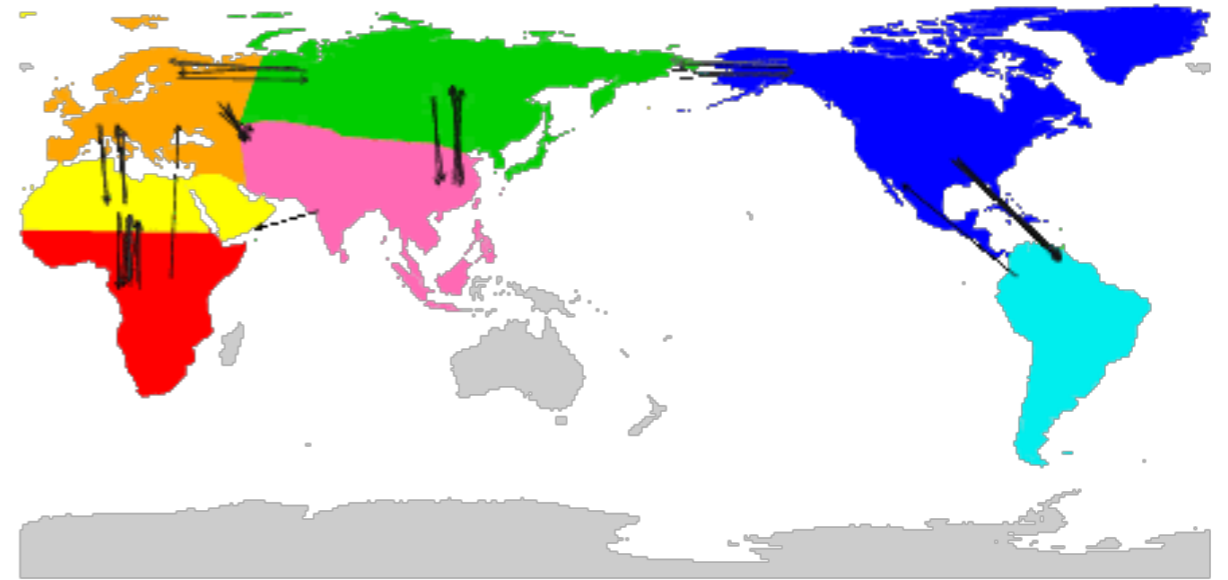
Conclusions

Not perfect, but useful

A “perfect” model would jointly sample everything

And use the biogeographical sampling model to influence the rate of fossil detection

Funded future work: New Zealand Marsden grant, with Alexei Drummond, Tanya Stadler, et al.



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DEC and DEC+J models fit the data?**

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**That means $8 \times 8 = 64$
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Calculate likelihoods for:

DEC, $d=0.2$, $e=0.1$

DEC+J, $d=0.2$, $e=0.1$, $j=0.15$

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sum of 64 DEC likes=7.0

sum of 64 DEC+J likes=7.0

Why 7?

8 states, 1 is null range

To get 1, times "base frequencies" of 1/7

**Validation #2 of
comparing DEC,
DEC+J likelihoods:**

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**These models are
special cases of
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**So: implement these models
in ClasSE and compare to
*BioGeoBEARS!***

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**ClASSE:
Cladogenetic
State-dependent
Speciation and
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**Set extinction to 0,
condition on
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ClasSE:

Cladogenetic

State-dependent

Speciation and ← **Yule process**

Extinction **(pure birth)**

Set extinction to 0,

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ClasSE:

Cladogenetic

State-dependent

Speciation and

Extinction

**To simplify to
DEC/DEC+J:**

**But with Yule rate
divided into**

**cladogenetic events
(rate * probability)**

**← Yule process
(pure birth)**

**Set extinction to 0,
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DEC, DEC+J (BioGeoBEARS) vs. ClaSSE (diversitree)

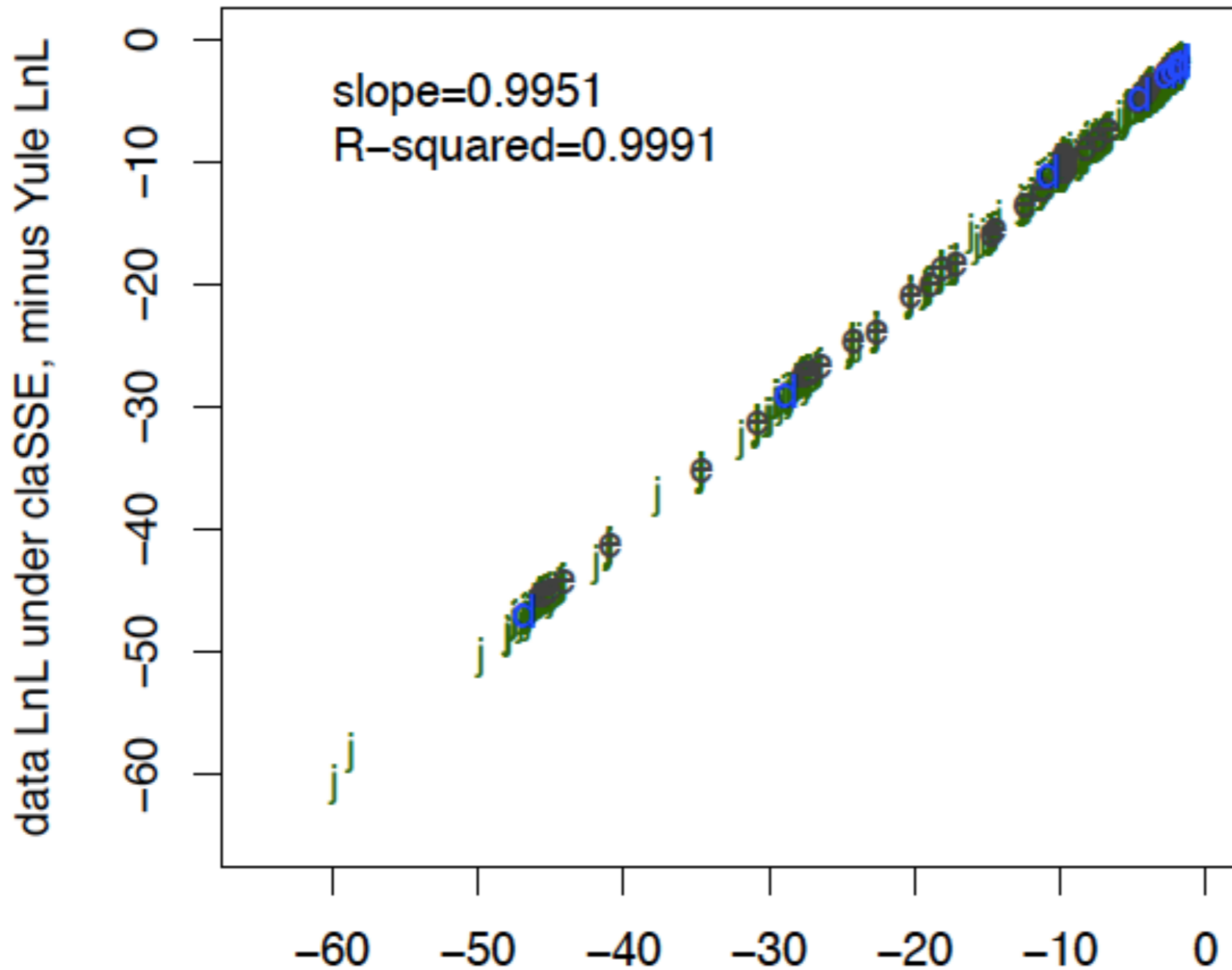
To make ClaSSE & DEC equivalent:

DEC or DEC+J equals ClaSSE, *if*:

- d and e control ClaSSE character change rates
- all extinction rates set to zero
- speciation rate for each cladogenesis =
(Yule rate) times weight/sum(weights)
- subtract equilibrium probabilities at the root

DEC, DEC+J (BioGeoBEARS) vs. ClaSSE (diversitree)

BioGeoBEARS log-likelihoods vs.
claSSE log-likelihoods (all)



LnLs for:

DEC-e = d

DEC = e

DEC+J = j

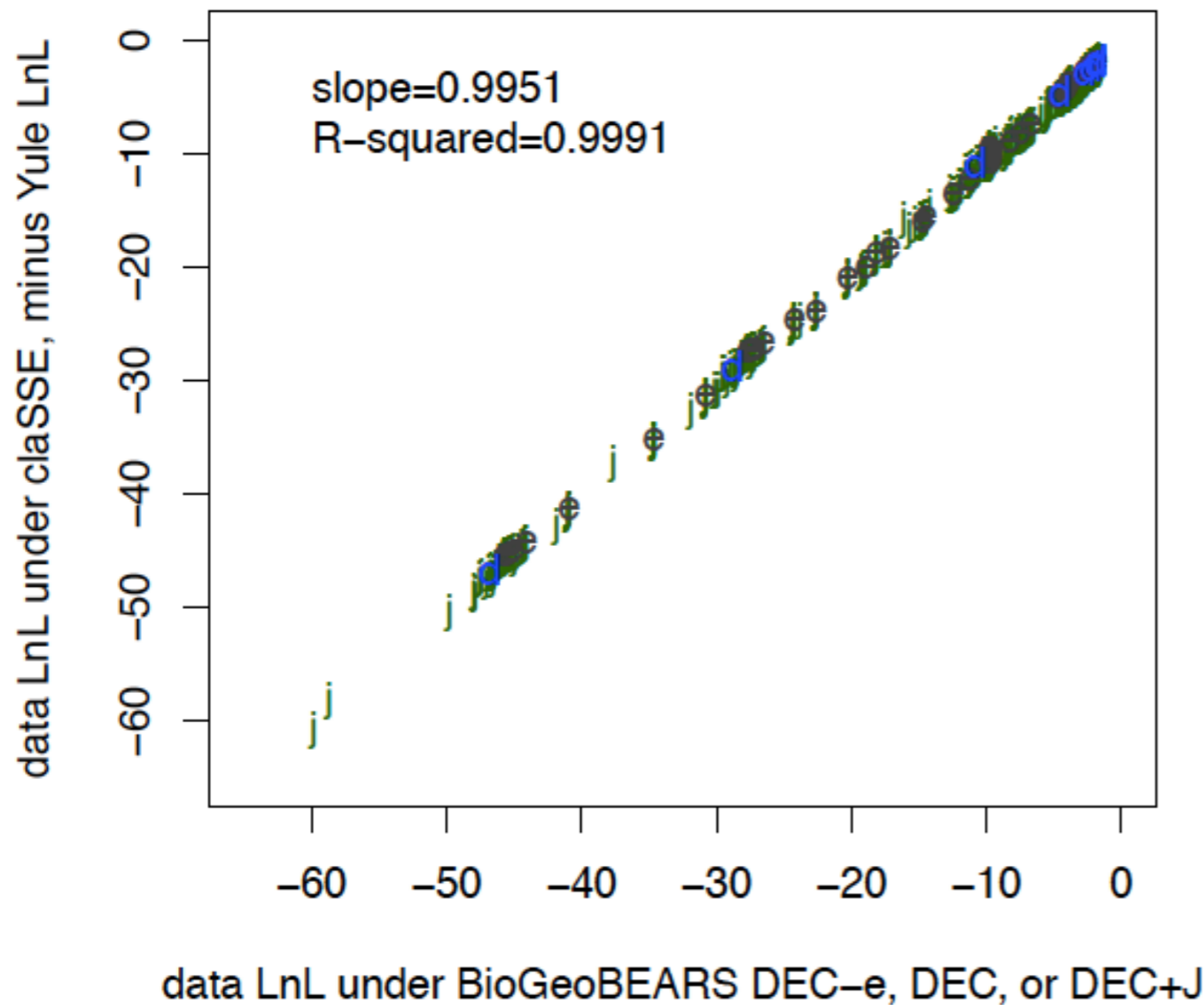
Summary:

Saying DEC and DEC+J are not statistically comparable...

...is identical to saying two submodels of ClaSSE are not comparable

...or that two valid conditional probabilities of the same data are not comparable

BioGeoBEARS log-likelihoods vs. claSSE log-likelihoods (all)



For these scripts, google “BioGeoBEARS validation”:

<http://phylo.wikidot.com/biogeobears-validation>